2

Units of measurement and applications

The main mathematical ideas in this chapter are:

► understanding approximations and significant figures
► expressing numbers in scientific notation
► converting between metric units of measurement
► using prefixes for units of measurement
► calculating error in measurement
► understanding accuracy of measurement
► calculating rates and ratios
► determining percentage change
► calculating the unitary method.

2A Significant figures

The **significant figures** in a number are the important or meaningful figures. A crowd of 61 348 is approximated to 61 000 to indicate that only the first 2 figures (digits) are important.

It is impossible to cut a piece of timber to a length of 1.333 333… m. The digits after the 4th figure, are completely meaningless in this case.

WORKED EXAMPLE 1

The first significant figure in a number is the first non-zero digit, reading from left to right. Round each of the following to:

i 1 significant figure   **ii** 2 significant figures   **iii** 3 significant figures.

a 293 568   **b** 0.076 04

		Solve	Think	Apply
a	**i**	300 000	The first non-zero digit is 2. This is the first significant figure. The next digit (9) is bigger than 5; thus, rounded to 1 significant figure, 293 568 ≈ 300 000. (This is the same as rounding 293 568 to the nearest 100 000, as the first significant figure is in the 100 000s column.)	Locate the relevant significant figure and then round appropriately using the next digit.
	ii	290 000	The second significant figure is 9. The next digit (3) is smaller than 5; thus, rounded to 2 significant figures, 293 568 ≈ 290 000. (This is the same as rounding 293 568 to the nearest 10 000, as the second significant figure is in the 10 000s column.)	
	iii	294 000	The third significant figure is 3. The next digit is 5; thus, rounded to 3 significant figures, 293 568 ≈ 294 000. (This is the same as rounding 293 568 to the nearest 1000, as the third significant figure is in the 1000s column.)	
b	**i**	0.08	The first non-zero digit is 7. This is the first significant figure. The next digit (6) is bigger than 5; thus, rounded to 1 significant figure, 0.076 04 ≈ 0.08. (This is the same as rounding 0.076 04 to 2 decimal places, as the first significant figure is in the second place after the decimal point, or to the nearest hundredth, as the first significant figure is in the hundredths column.)	
	ii	0.076	The second significant figure is 6. The next digit (0) is smaller than 5; thus, rounded to 2 significant figures, 0.076 04 ≈ 0.076. (This is the same as rounding 0.076 04 to 3 decimal places, as the second significant figure is in the third place after the decimal point, or to the nearest thousandth, as the second significant figure is in the thousandths column.)	
	iii	0.0760	The third significant figure is 0. The next digit (4) is smaller than 5; thus, rounded to 3 significant figures, 0.076 04 ≈ 0.0760. (This is the same as rounding 0.076 04 to 4 decimal places, as the third significant figure is in the fourth place after the decimal point, or to the nearest ten-thousandth.)	

MEASUREMENT

EXERCISE 2A

1 Complete the following to round:
 a 5368 to 3 significant figures.
  The first non-zero digit is ____. This is the first significant figure. The third significant figure is ____.
  The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer
  to ____ than to ____. Thus, rounded to 3 significant figures, 5368 ≈ ____.
 b 0.062 53 to 3 significant figures.
  The first non-zero digit is ____. This is the first significant figure. The third significant figure is ____.
  The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer
  to ____ than to ____. Thus, rounded to 3 significant figures, 0.062 53 ≈ ____.

2 Round the following to 1 significant figure.
 a 42 600   b 59   c 4.6   d 108
 e 0.6529   f 0.0082   g 0.025   h 990

3 Round the following to 2 significant figures.
 a 290 365   b 3960   c 24.9   d 2653
 e 8.63   f 0.0487   g 0.000 162 8   h 0.003 97

4 Round the following to 3 significant figures.
 a 3688   b 20 657   c 154 299   d 813.4
 e 14.294   f 0.003 508 1   g 0.039 14   h 1.999

5 Round the following to:
   i 1 significant figure   ii 2 significant figures   iii 3 significant figures.
 a 17.256   b 0.450 72   c 521 500   d 8.045   e 0.002 095

2B Scientific notation

Scientific (or standard) **notation** is a convenient way of writing very large and very small numbers. A number
written in scientific notation is written as the product of a number between 1 and 10 and a power of 10; that is,
it is put in the form $A \times 10^n$ where A lies between 1 and 10, and n is an integer (whole number).

14F
14G
14H
14J

WORKED EXAMPLE 1

State whether the following numbers are expressed in scientific notation.
 a 5.3×10^7 b 78×10^5 c $4.9 \times 10\,000$ d 3×10^{-4} e 294 000

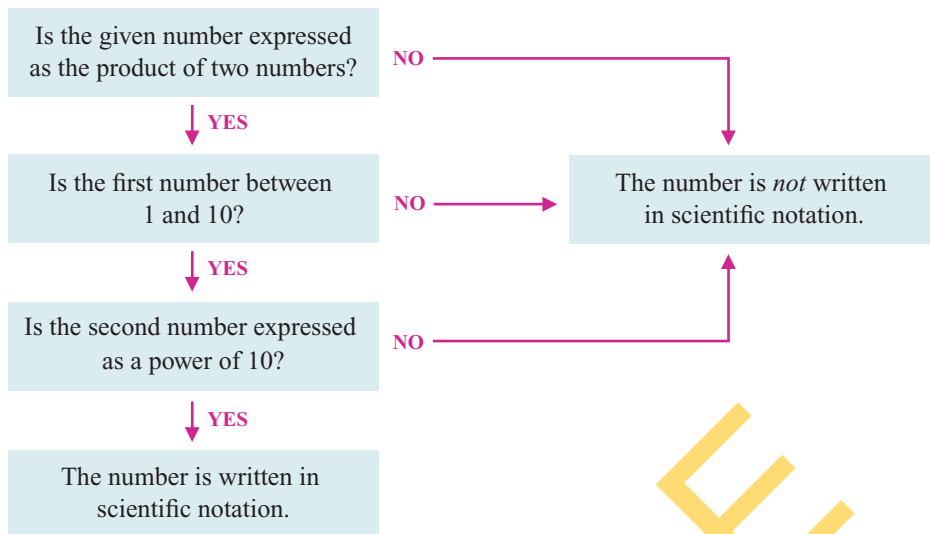|	Solve	Think	Apply
a	Yes	The first number in the product (5.3) is between 1 and 10, the second number (10^7) is a power of 10.	A number is written in scientific notation if it is written as the product of a number between 1 and 10 and a power of 10.
b	No	The first number (78) is not between 1 and 10.	
c	No	The second number (10 000) is not written as a power of 10.	
d	Yes	The first number in the product (3) is between 1 and 10, the second number (10^{-4}) is a power of 10.	
e	No	294 000 is not written as a product of two numbers.	

MEASUREMENT

EXERCISE 2B

1 Use the flow diagram below to determine whether the given numbers are expressed in scientific notation.

a 2.91×10^{-17} **b** 53×10^6 **c** $3.8 \times 100\,000$ **d** 3^{26}

Is the given number expressed as the product of two numbers? — **NO** →

↓ **YES**

Is the first number between 1 and 10? — **NO** →

↓ **YES**

Is the second number expressed as a power of 10? — **NO** →

↓ **YES**

The number is written in scientific notation.

The number is *not* written in scientific notation.

2 State whether the following numbers are written in scientific notation.

a 3.6×10^5 **b** $5.2 \times 10\,000$ **c** 21×10^5 **d** 2.87×10^{-6}

e $6.07 \times \frac{1}{1\,000\,000}$ **f** 594×10^{-5} **g** 70×10^8 **h** 3.06×10^{-9}

WORKED EXAMPLE 2

Write these numbers in scientific notation.

a $138\,000$ **b** $0.000\,486$

	Solve	Think	Apply
a	1.38×10^5	Move the decimal point so that it is positioned between the first and second digits. This always produces a number between 1 and 10, in this case 1.380 00. Count the number of places back to the original position of the decimal point: $1.380\,00$ Number of places = 5 to the right $= +5$ This becomes the power of 10: $138\,000 = 1.380\,00 \times 10^5$ $= 1.38 \times 10^5$ (leave off the zeros)	Move the decimal point so that it is positioned between the first two digits. This produces a number between 1 and 10. Count the number of places back to the original position of the decimal point. This becomes the power of 10. *Note:* When counting back to the original position of the decimal point, counting to the right produces a positive power of 10 and counting to the left produces a negative power of 10.
b	4.86×10^{-4}	Move the decimal point so that it is positioned between the first and second digits, in this case 4.86. Count the number of places back to the original position of the decimal point: $0\,0004.86$ Number of places = 4 to the left $= -4$ This becomes the power of 10: $0.000\,486 = 4.86 \times 10^{-4}$	

MEASUREMENT

3 Complete the following to write the number in scientific notation.

a 243 000

Position the decimal point between the first two digits → _____.

The number of places to the original position of the decimal point = _____ to the _____

$$= +\underline{\hspace{2em}}.$$

Hence, 243 000 = _____ $\times 10^{\square}$

b 0.000 586

Position the decimal point between the first two digits → _____.

The number of places to the original position of the decimal point = _____ to the _____

$$= -\underline{\hspace{2em}}.$$

Hence, 0.000 586 = _____ $\times 10^{\square}$

4 Use the method of Worked Example 2 to write each number in scientific notation.

a 526 000	**b** 28 000	**c** 7 000 000	**d** 49 800
e 28 000 000	**f** 603 000 000	**g** 910 000	**h** 13 200 000 000

5 Use the method of Worked Example 2 to write each number in scientific notation.

a 0.000 43	**b** 0.008 21	**c** 0.000 007	**d** 0.000 029
e 0.065	**f** 0.000 387	**g** 0.000 008 2	**h** 0.000 06

WORKED EXAMPLE 3

Write the following as ordinary (or basic) numbers.

a 4.83×10^7 **b** 9.2×10^{-6}

	Solve	Think	Apply
a	48 300 000	As the power of 10 is $+7$, the decimal point is moved 7 places to the right. 48 300 000 so $4.83 \times 10^7 = 48\ 300\ 000$	The magnitude of the power of 10 tells us how many places to move the decimal point. If the power of 10 is positive, move the decimal point to the right. If the power is negative, move it to the left.
b	0.000 009 2	As the power of 10 is -6, the decimal point is moved 6 places to the left. 0 000 009 2 so $9.2 \times 10^{-6} = 0.000\ 009\ 2$	

6 Complete the statements to write each number as an ordinary number.

a 5.48×10^6

Move the decimal point ____ places to the ____. Hence, $5.48 \times 10^6 =$ _____.

b 3.09×10^{-5}

Move the decimal point ____ places to the ____. Hence, $3.09 \times 10^{-5} =$ _____.

7 Express these scientific numbers as ordinary numbers.

a 3.4×10^6	**b** 8.3×10^8	**c** 2.94×10^7	**d** 2.58×10^5
e 5.26×10^5	**f** 3.02×10^{12}	**g** 2.9×10^7	**h** 8.75×10^8

8 Write the basic number for:

a 5.9×10^{-4}	**b** 3.2×10^{-6}	**c** 7.1×10^{-8}	**d** 2×10^{-3}
e 8×10^{-7}	**f** 2.64×10^{-5}	**g** 8.67×10^{-9}	**h** 2.97×10^{-6}

MEASUREMENT
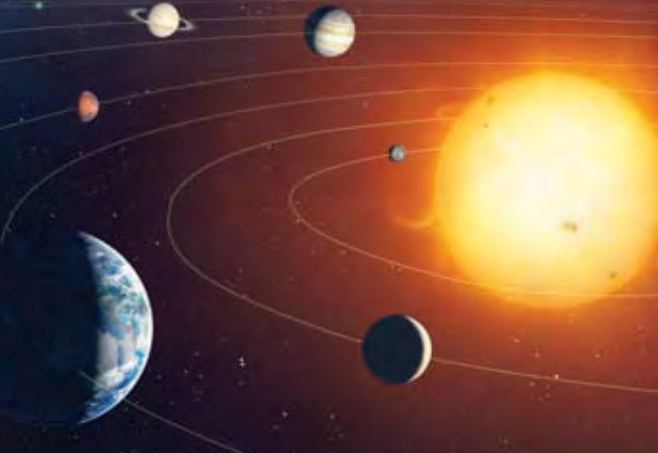
9 Express the following numbers in scientific notation.

a The distance of Mars from the Sun is approximately 229 000 000 km.

b The diameter of the hydrogen atom is 0.000 000 000 025 4 m.

c The Sun produces the same amount of light as 3 000 000 000 000 000 000 000 000 000 candles.

d There are approximately 130 000 hairs on a person's head.

e There are approximately 10 000 000 000 000 cells in the human body.

10 Change the following to ordinary numbers.

a There are approximately 3.16×10^7 s in a year.

b The number of different hands of the card game Poker is approximately 2.6×10^6.

c Swarms of locusts have been known to contain as many as 3×10^{10} locusts.

d The size of the influenza virus is approximately 2.6×10^{-4} mm.

e A molecule's diameter is 8.9×10^{-7} mm.

WORKED EXAMPLE 4

Use your calculator to find:

a $(3.5 \times 10^7) \times (2.4 \times 10^9)$

b $(6.4 \times 10^8) \div (2.5 \times 10^{-6})$

c $\sqrt{2.4 \times 10^{10}}$

d $(1.5 \times 10^7)^3$

	Solve	Think	Apply
a	8.4×10^{16}	Possible steps using a Casio calculator are: Press 3.5 [EXP] 7 [×] 2.4 [EXP] 9 [=] *Answer*: 8.4×10^{16}	If the answer is not displayed in scientific notation, you could use the SCI function on the calculator to express the answer in this form.
b	2.56×10^{14}	Press 6.4 [EXP] 8 [÷] 2.5 [EXP] -6 [=] *Answer*: 2.56×10^{14}	
c	1.55×10^5	Press [√] 2.4 [EXP] 10 [=] *Answer*: 1.55×10^5 to 3 significant figures	
d	3.375×10^{21}	Press 1.5 [EXP] 7 [^] 3 [=] *Answer*: 3.375×10^{21}	

11 Calculate the following correct to 3 significant figures. Give the answer in scientific notation.

a $(2.6 \times 10^8) \times (4.1 \times 10^7)$

b $(5.8 \times 10^9) \times (8.2 \times 10^{12})$

c $(9.2 \times 10^{24}) \times (3.5 \times 10^{-8})$

d $(5.8 \times 10^{-6}) \times (2.4 \times 10^{-9})$

e $(8.4 \times 10^{18}) \div (2.5 \times 10^7)$

f $(5.25 \times 10^{12}) \div (4.2 \times 10^{-8})$

g $(1.82 \times 10^{-6}) \times (2.9 \times 10^{-10})$

h $\sqrt{5.76 \times 10^{16}}$

i $\sqrt{6.8 \times 10^{17}}$

j $\sqrt{(3.1 \times 10^8)^5}$

k $(8 \times 10^{-10})^6$

l $(2.8 \times 10^8)^4 \times (1.6 \times 10^6) \div (2.1 \times 10^{15})$

12 Light travels at a velocity of 300 000 km/s.
  a Express this number in scientific notation.
  b How far does light travel in:
    i 1 minute?   **ii** 1 hour?
    iii 1 day?   **iv** 1 year?
  c If light takes 4.1 min to reach Earth from Mars, what is the distance from Earth to Mars?

13 The radius of the Earth is approximately 6400 km.
  a Calculate the area of the Earth's surface, to 2 significant figures. (Use $A = 4\pi r^2$.)
  b Calculate the volume of the Earth, to 2 significant figures. (Use $V = \frac{4}{3}\pi r^3$.)

14 The radius of the Earth's orbit around the Sun is approximately 1.49×10^8 km. Assuming that the orbit is circular, calculate the distance travelled by the Earth in one orbit, to 2 significant figures. (Use $C = 2\pi r$.)

15 The human brain contains about 10^{10} cells.
  a Write this as an ordinary number.
  b Each of these cells is about 2.8×10^{-5} m long. If all the brain cells could be placed next to each other, in a straight line, how long would this line be?

16 Measure your pulse to determine the number of times your heart beats in a minute. If you live to 75 years of age, how many times will your heart beat in this time? (Assume your pulse remains constant.)

2C Metric units of measurement

14G
14H

The metric system of measurement uses base units for quantities such as length, mass, capacity, area and volume. The prefix indicates the factor of 10 by which the base unit is multiplied.

RESEARCH PROJECT 2.1

EXERCISE **2C**

1 Complete this conversion diagram for length.

	× 1000 →		____ →		× 10 →	
kilometres (km)		metres (m)		centimetres (cm)		millimetres (mm)

↑ ____ ↑ ____ ↑ ÷ 10
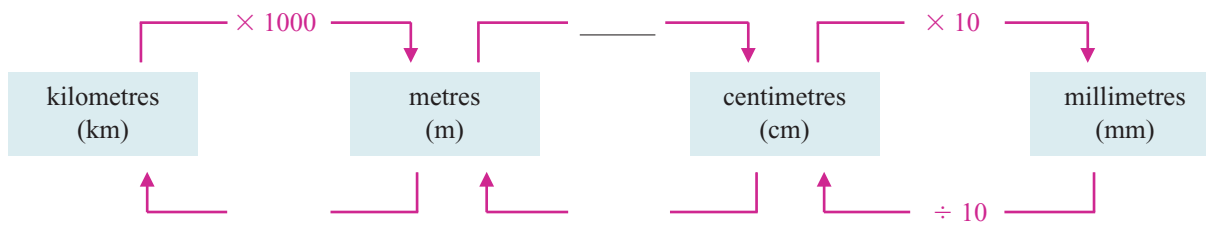
2 Convert these lengths.
  a 3.6 km to m   **b** 8.4 m to mm   **c** 34.82 m to cm   **d** 0.56 km to m
  e 2.9 m to cm   **f** 0.964 m to mm   **g** 0.658 m to cm   **h** 45.2 cm to mm
  i 15.68 km to m   **j** 3.69 cm to mm   **k** 16.37 m to mm   **l** 4.265 km to m

3 **a** **i** Complete: 1 km = ____ cm.   **ii** Express the answer in scientific notation.
   b **i** Complete: 1 km = ____ mm.   **ii** Express the answer in scientific notation.

MEASUREMENT

4 Convert these lengths.

   a 7000 m to km   **b** 594 cm to m   **c** 8930 m to km   **d** 6000 mm to m

   e 40 mm to cm   **f** 85 m to km   **g** 800 cm to m   **h** 328 mm to cm

   i 620 mm to m   **j** 14 300 m to km   **k** 86 cm to m   **l** 630 m to km

   m 94 mm to cm   **n** 70 mm to m   **o** 24 895 m to km   **p** 23 000 mm to m

   q 14 960 mm to m   **r** 16 270 cm to m   **s** 3600 cm to m   **t** 72 945 mm to m

5 What would be a convenient unit (mm, cm, m, km) to use to measure the following?

   a width of the classroom   **b** length of a textbook

   c height of a student   **d** length of a baby

   e length of your foot   **f** length of a match

   g distance from Sydney to Brisbane

   h length of material for a dress

   i length of a driveway

   j distance between railway stations

6 Complete this conversion diagram for mass.

	× 1000 →		_____ →		× 1000 →	
tonnes (t)		kilograms (kg)		grams (g)		milligrams (mg)
	← _____		← ÷ 1000		← _____	
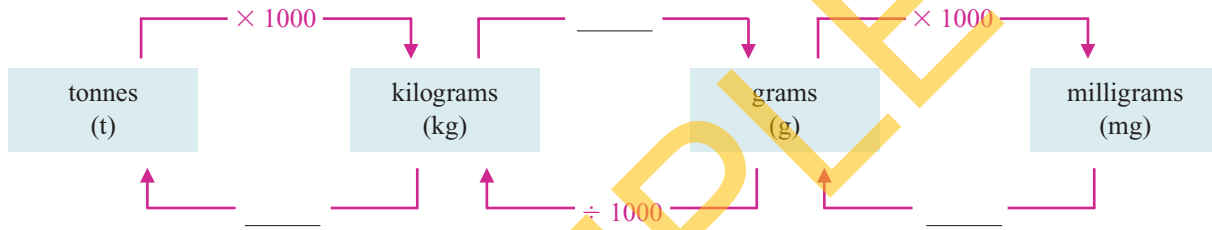
7 Convert these masses.

   a 2.7 t to kg   **b** 4.5 g to mg   **c** 23.92 kg to g   **d** 0.34 t to kg

   e 5.6 kg to g   **f** 1.758 t to kg   **g** 23.49 t to kg   **h** 0.8 g to mg

   i 4.05 kg to g   **j** 0.875 kg to g   **k** 2.05 t to kg   **l** 0.05 g to mg

8 a i Complete: 1 t = _____ g.   **ii** Write the answer in scientific notation.

   b i Complete: 1 t =_____ mg.   **ii** Write the answer in scientific notation.

   **c i** Complete: 1 kg = _____ g.   **ii** Write the answer in scientific notation.

9 Convert the following.

   a 4000 kg to t   **b** 8500 g to kg   **c** 1650 mg to g   **d** 750 g to kg

   e 45 mg to g   **f** 1480 g to kg   **g** 400 kg to t   **h** 950 mg to g

   i 90 g to kg   **j** 9 mg to g   **k** 5 kg to t   **l** 200 g to kg

10 State a convenient unit to use (t, kg, g, mg) to measure the mass of the following.

   a a man   **b** a jar of jam   **c** a packet of biscuits   **d** a bag of sand

   e a truck   **f** an elephant   **g** a vitamin pill

   h a knitting needle   **i** a paperclip

11 Complete this conversion diagram for capacity.

	× 1000 →		_____ →	
kilolitres (kL)		litres (L)		millilitres (mL)
	← _____		← ÷ 1000	
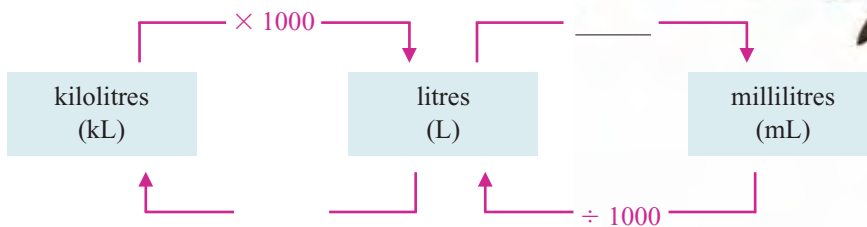
12 Convert the following.

   a 35 kL to L   **b** 15.9 L to mL
   c 1.65 L to mL   **d** 0.85 kL to L
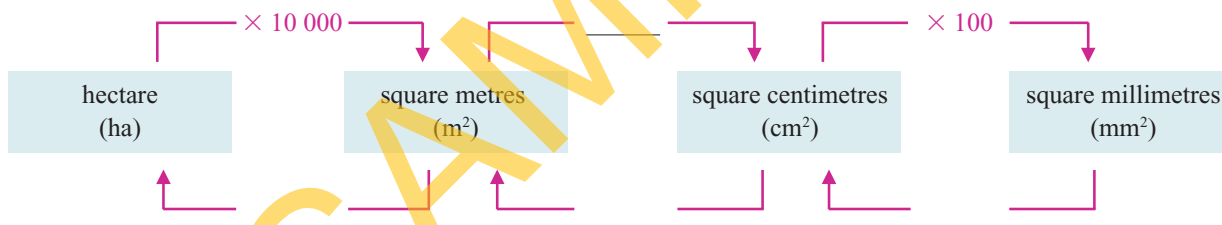   **e** 0.06 L to mL   **f** 1.08 kL to L
   g 0.015 L to mL   **h** 0.005 kL to L

13 How many millilitres are there in 1 kL?
Express your answer in scientific notation.

14 Convert the following.

   a 15 000 mL to L   **b** 8000 L to kL
   c 7600 mL to L   **d** 800 mL to L
   e 9280 L to kL   **f** 725 L to kL
   g 95 mL to L   **h** 40 L to kL

15 State an appropriate unit to use (kL, L, mL) to measure the capacity of a:

   a teaspoon   **b** swimming pool
   c bucket   **d** fish tank
   e laundry tub   **f** farm dam
   g car's petrol tank **h** kettle

16 Complete this conversion diagram for area.

hectare (ha)	square metres (m^2)	square centimetres (cm^2)	square millimetres (mm^2)

$\times 10\,000$ $\times 100$

17 Convert these areas.

   a 2.6 ha to m^2 **b** 4.9 m^2 to cm^2 **c** 14 cm^2 to mm^2 **d** 0.752 m^2 to cm^2
   e 1.65 ha to m^2 **f** 24.8 cm^2 to mm^2 **g** 8.294 ha to m^2 **h** 5.671 m^2 to cm^2

18 Convert the following.

   a 63 000 m^2 to ha   **b** 127 000 cm^2 to m^2 **c** 810 mm^2 to cm^2
   d 45 680 cm^2 to m^2 **e** 298 000 m^2 to ha   **f** 2400 mm^2 to cm^2

19 Complete this conversion diagram for volume.

cubic metres (m^3)	cubic centimetres (cm^3)	cubic millimetres (mm^3)

$\times 1\,000\,000$ $\times 1000$
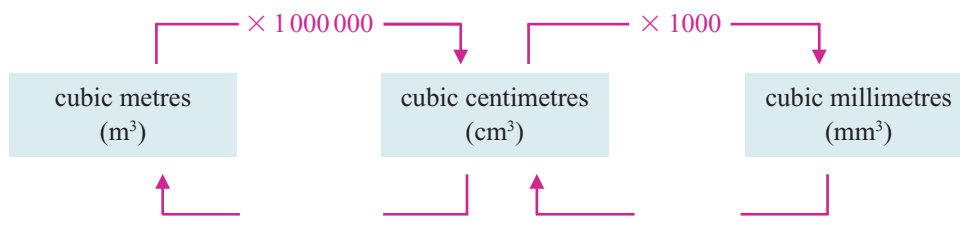
20 Convert these volumes.

   a 3.9 m^3 to cm^3 **b** 25.6 cm^3 to mm^3 **c** 0.64 m^3 to cm^3 **d** 7.39 m^3 to mm^3

21 Convert the following.

a 7 400 000 cm³ to m³   **b** 56 700 mm³ to cm³   **c** 690 000 cm³ to m³

22 Complete the conversion diagram for time.

```
        ┌──── × 24 ────┐      ┌──── ____ ────┐      ┌──── ____ ────┐
        │              ▼      │              ▼      │              ▼
    ┌────────┐     ┌────────┐     ┌────────┐     ┌────────┐
    │  days  │     │ hours  │     │minutes │     │seconds │
    │  (d)   │     │  (h)   │     │ (min)  │     │  (s)   │
    └────────┘     └────────┘     └────────┘     └────────┘
        ▲              │      ▲              │      ▲              │
        └──── ____ ────┘      └──── ____ ────┘      └──── ÷ 60 ────┘
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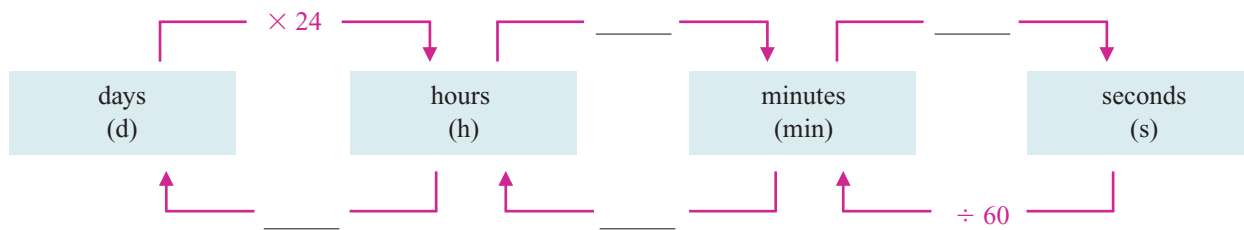
23 Convert these times.

a 3 d to h   **b** 5 d to h   **c** 4 h to min   **d** 7 min to s

e 15 min to s   **f** 2 d to min   **g** 3 h to s   **h** 12 h to s

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WORKED EXAMPLE 1

Convert the following to minutes.

a 2 h and 25 min   **b** 2.25 h

	Solve	Think	Apply
a	145 min	2 h and 25 min = 2 × 60 + 25 min = 145 min	Multiply the number of hours by 60 and add on any extra minutes.
b	135 min	2.25 h = 2.25 × 60 min = 135 min	

---

24 Complete these conversions.

a 2 d and 7 h = 2 × ____ + ____ h
     = ____ h

b 7 h and 25 min = 7 × ____ + ____ min
     = ____ min

c 5 min and 13 s = 5 × ____ + ____ s
     = ____ s

d 3.75 d = 3.75 × ____ h
     = ____ h

e 4.8 h = 4.8 × ____ min
     = ____ min

f 7.2 min = 7.2 × ____ s
     = ____ s

25 Convert the following.

a 3 h and 26 min to min   **b** 5 h and 51 min to min   **c** 2 min and 16 s to s

d 12 min and 17 s to s   **e** 2 d and 8 h to h   **f** 5 d and 23 h to h

g 1 h, 5 min and 40 s to s   **h** 4 h, 38 min and 18 s to s **i** 1 d, 4 h and 25 min to min

j 3 d, 16 h and 50 min to min **k** 4.6 h to min   **l** 2.4 h to min

m 3.25 h to min   **n** 5.2 min to s   **o** 1.9 min to s

p 7.8 min to s   **q** 4.75 d to h   **r** 2.375 d to h

s 5.82 h to s   **t** 5.24 h to s   **u** 3.25 d to h

Convert 384 min to:

a hours   **b** hours and minutes.

	Solve	Think	Apply
a	6.4 h	384 min = 384 ÷ 60 h = 6.4 h	Divide the number of minutes by 60.
b	6 h 24 min	0.4 h = 0.4 × 60 min = 24 min Hence, 6.4 h = 6 h and 24 min. *Calculator method* Find the degrees/minutes/seconds key. Press 6.4 SHIFT ◦ ′ ″ The calculator displays 6°24°0° which may be read as 6 h and 24 min.	If the answer contains a decimal, either use the degrees/minutes/seconds key or multiply the decimal part by 60 to convert the answer to hours and minutes.

26 Complete the following conversions.

  a   114 h = 114 ÷ _____ d   **b**   456 min = 456 ÷ _____ h   **c**   291 s = 291 ÷ _____ min

            = _____ d   = _____ h   = _____ min

                                                                                            = 5.7 h

  d   342 min = 342 ÷ _____ h   **e**   438 s = 438 ÷ _____ min

      Now 0.7 h = 0.7 × _____ min   = 7.3 min

                 = _____ min   Now 0.3 min = 0.3 × _____ s

      ∴ 342 min = 5 h and _____ min   = _____ s

                                              ∴ 438 s = 7 min and _____ s

27 Convert the following to:

    i hours   **ii** hours and minutes.

  a 168 min   **b** 192 min   **c** 261 min   **d** 339 min   **e** 267 min

28 Convert the following to:

    i minutes   **ii** minutes and seconds.

  a 114 s   **b** 216 s   **c** 153 s   **d** 411 s   **e** 243 s

INVESTIGATION 2.1

2D Prefixes for units of measurement

Prefixes indicate the factor of 10 by which the base metric unit is multiplied. For example, the unit kilogram uses the base unit for mass, the gram, with the prefix *kilo*, which indicates a multiplying factor of 1000: 1 kg = 1000 g.

The table at the top of the next page summarises the most common prefixes used for very large and very small measurements.

Prefix	Multiplying factor
tera (T)	$10^{12} = 1\,000\,000\,000\,000$
giga (G)	$10^9 = 1\,000\,000\,000$
mega (M)	$10^6 = 1\,000\,000$
kilo (k)	$10^3 = 1000$
centi (c)	$10^{-2} = 0.01$
milli (m)	$10^{-3} = 0.001$
micro (µ)	$10^{-6} = 0.000\,001$
nano (n)	$10^{-9} = 0.000\,000\,001$

WORKED EXAMPLE 1

Convert the following to metres.

a   3.6 Gm

b   7 µm

	Solve	Think	Apply
a	3.6 Gm $= 3.6 \times 10^9$ m or 3 600 000 000 m	1 Gm $= 1 \times 10^9$ m	Apply the multiplying factor for the prefix.
b	7 µm $= 7 \times 10^{-6}$ m or 0.000 007 m	1 µm $= 1 \times 10^{-6}$ m	

EXERCISE 2D

1 Convert these to metres.
   a   5.7 Mm
   b   9 cm
   c   8 nm

2 Convert these to grams.
   a   8 Gg
   b   4.2 mg
   c   5 µg

3 Convert these to litres.
   a   3 TL
   b   2.8 ML
   c   7 mL

WORKED EXAMPLE 2

Convert 5.6 m to:

a   km

b   µm

	Solve	Think	Apply
a	5.6 m $= \dfrac{5.6}{10^3}$ km $= 5.6 \times 10^{-3}$ or 0.0056 km	Divide 5.6 by the number of metres in a kilometre. $\dfrac{1}{10^3} = 10^{-3}$	Divide by the number of metres in the required unit.
b	5.6 m $= \dfrac{5.6}{10^{-6}}$ µm $= 5.6 \times 10^6$ or 5 600 000 µm	Divide 5.6 by the number of metres in a micrometre. $\dfrac{1}{10^{-6}} = 10^6$	

4 Convert 4.9 m to:

   a km   **b** nm

5 Convert 2.4 g to:

   a Mg   **b** μg

6 Convert 6.5 L to:

   a ML   **b** mL

WORKED EXAMPLE 3

Convert the following.

a 3.56 Tg to Mg   **b** 9.4 mg to μg

	Solve	Think	Apply
a	$3.56 \text{ Tg} = \dfrac{3.56 \times 10^{12}}{10^6} \text{ Mg}$ $= 3.56 \times 10^6 \text{ or } 3\,560\,000 \text{ Mg}$	Convert 3.56 Tg into g and divide by the number of grams in a megagram.	Convert to grams and divide by the number of grams in the required unit.
b	$9.4 \text{ mg} = \dfrac{9.4 \times 10^{-3}}{10^{-6}} \text{ μg}$ $= 9.4 \times 10^3 \text{ or } 9400 \text{ μg}$	Convert 9.4 mg into g and divide by the number of grams in microgram.	

7 Convert the following.

   a 7.2 Gm into Mg   **b** 2.9 cg into ng   **c** 8 Tm into km   **d** 4.3 cm into μm

   e 8.8 TL into ML   **f** 9 kL into mL   **g** 5.3 ms into ns   **h** 1.2 μs into ns

8 **a** Light travels approximately 9.46×10^{15} m in a year. Express this in terametres.

   b Warragamba dam holds approximately 2 580 000 ML of water at full capacity. How many gigalitres is this?

   c The distance from Mars to the Sun is 0.228 Tm. Convert this distance to kilometres.

   d The mass of a hydrogen atom is 1.67×10^{-24} g. What is the mass in nanograms of 1 million hydrogen atoms?

   e A computer can access its memory in 24 ns. Convert this to microseconds.

2E   Error in measurement

When physically measuring a quantity there are several sources of possible error and uncertainty.

- Errors occur if the zero on the scale of the measuring instrument does not coincide with the end of the object or with the pointer on the measuring instrument.

- An error occurs if the end of the measuring instrument has been damaged. In this case start measuring from the 1, say, instead of 0.

- Parallax error occurs if your eye is not directly above the scale on the measuring instrument.

- Calibration error can occur if the scale is not accurately marked on the measuring instrument.

- There is always an error due to the limit of reading the measuring instrument.
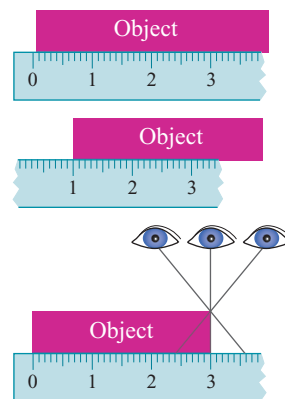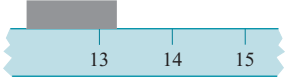
Repeating a measurement a number of times and averaging the values can reduce the effect of any errors.

MEASUREMENT

WORKED EXAMPLE 1

John measured the width of his maths textbook five times using a ruler marked in millimetres. The results were 190 mm, 189 mm, 190 mm, 192 mm and 190 mm. Average these measurements to give an approximation for the width of the book, to the nearest millimetre.

Solve	Think	Apply
Average $= \dfrac{190 + 189 + 190 + 192 + 190}{5}$ $= 190.2$ mm $= 190$ mm, to the nearest mm	Determine the average by finding the sum of the measurements and then dividing by the number of measurements. The answer is 190 mm, to nearest mm, as 190.2 is closer to 190 than to 191.	Averaging measurements reduces the effects of any errors. The answer should be given to the same degree of accuracy as the given measurements (in this case, to the nearest millimetre). The differences in the measurements could have been caused by any of the errors discussed above.

EXERCISE 2E

1 A student measured the length of his textbook using a ruler marked in millimetres. The results were 256 mm, 255 mm, 255 mm, 254 mm and 254 mm. Average these measurements to give an approximation for the length of the book, to the nearest millimetre.

2 Average the following to give an approximation for the true measurement. In each case, the measurements were taken to the same degree of accuracy.
  a 83 mm, 85 mm, 84 mm, 85 mm, 85 mm, 84 mm   **b** 4.9 kg, 4.8 kg, 4.9 kg, 5.0 kg, 4.9 kg
  c 162 mL, 162 mL, 160 mL, 161 mL, 161 mL, 162 mL   **d** 22.49 s, 22.61 s, 22.54 s, 22.56 s, 22.52 s

3 Have five students measure your height to the nearest centimetre. Average these measurements to give an approximation for your true height.

4 The diagrams show several steel rods being measured with a ruler divided into centimetres. Write the length of each rod, using the scale given on the ruler.

  a   13   14   15   **b**   13   14   15

  c   13   14   15   **d**   13   14   15

5 The length of a rod is measured using the ruler in question **4**, and the measurement is recorded as 14 cm.
  a Would this be the exact length of the rod?
  b Between what values would the actual length lie?
  c What is the greatest possible error in stating that the length is 14 cm?
  d How could we find a more accurate value of the length of the rod?

Insight Mathematics General 11

MEASUREMENT

Greatest possible error and percentage error

The rod in question **5** has been measured to the nearest centimetre because this is the smallest unit on the ruler: the length is closer to 14 cm than to 13 cm or 15 cm. The greatest possible error is 0.5 cm or half of the smallest scale (cm) on the ruler.

The actual length will lie between 13.5 cm and 14.5 cm; that is, between $14 - 0.5$ cm and $14 + 0.5$ cm. To obtain a more accurate measurement, we would need to use a more accurate ruler, one that has smaller units on it.

> The smallest unit on a measuring instrument is called the **limit of reading** of the instrument.
> The **greatest possible error** (**GPE**) in measuring a quantity (sometimes called the **absolute error**) is equal to half the limit of reading.
> The smallest and largest values between which the actual measurement lies are called the **lower** and **upper limits** of the **true measurement**.

As there is always some degree of error in a numerical value found by measurement, it follows that the results of any calculations involving this value will also contain a degree of error.

WORKED EXAMPLE 2

For each of the measurements below, find:
    i the smallest unit of measurement (the limit of reading)
    ii the greatest possible error (GPE).
a 18 cm   **b** 2.4 kg

		Solve	Think	Apply
a	**i**	The smallest unit of measurement is 1 cm; that is, the measurement has been made to the nearest centimetre. Hence: Limit of reading = 1 cm	The last significant figure of the number is in the units column. Hence, the smallest scale on the measuring instrument is 1 cm; that is, the measurement has been made to the nearest centimetre, the limit of reading.	The position of the last digit in the number determines the smallest scale on the measuring instrument used. This is the limit of reading of the instrument. The greatest possible error is half the limit of reading.
	ii	GPE $= \frac{1}{2} \times 1$ cm $= 0.5$ cm	GPE $= \frac{1}{2} \times$ limit of reading	
b	**i**	The smallest unit of measurement is 0.1 kg; that is, the measurement has been made to the nearest 0.1 of a kilogram. Hence: Limit of reading = 0.1 kg	The last significant figure of the number is in the tenths column. Hence, the smallest scale on the measuring instrument is 0.1 kg; the measurement has been made to the nearest 0.1 of a kilogram. This is the limit of reading.	
	ii	GPE $= \frac{1}{2} \times 0.1$ kg $= 0.05$ kg	GPE $= \frac{1}{2} \times$ limit of reading	

6 Complete the following.

    a For a measurement given as 138 cm, the last significant figure is in the _____ column.
    Hence, the smallest scale on the measuring instrument is _____.
    The measurement has been made to the nearest _____.
    Hence, the limit of reading = _____.
    GPE $= \frac{1}{2} \times$ _____ = _____

b For a measurement given as 11.7 s, the last significant figure is in the _____ column.
Hence, the smallest scale on the measuring instrument is _____.
The measurement has been made to the nearest _____.
Hence, the limit of reading = _____.
$GPE = \frac{1}{2} \times$ _____ = _____

7 For each of the following measurements, find:
    i the smallest unit of measurement (the limit of reading)
   ii the greatest possible error (GPE).

  a 16 cm   **b** 286 g   **c** 38 m   **d** 16 L
  e 3.6 kg   **f** 15.3 s   **g** 2.8 L   **h** 3.76 m

WORKED EXAMPLE 3

For each of the following measurements, find:
    i the smallest unit of measurement (the limit of reading)   **ii** the absolute error
   iii the lower and upper limits of the true measurement.

  a 16 s   **b** 9.38 m

		Solve	Think	Apply
a	**i**	The smallest unit of measurement is 1 s; that is, this measurement of time has been made to the nearest second. Limit of reading = 1 s	Find the limit of reading (1 s) and the absolute error (0.5 s) as in Worked Example 2. Lower limit = 16 s − 0.5 s Upper limit = 16 s + 0.5 s	Find the limit of reading and the greatest possible error. Lower limit = measurement − absolute error Upper limit = measurement + absolute error
	ii	Absolute error = $\frac{1}{2} \times 1 = 0.5$ s		
	iii	Lower limit = 16 − 0.5 = 15.5 s Upper limit = 16 + 0.5 = 16.5 s True measurement is between 15.5 and 16.5 s.		
b	**i**	Smallest unit of measurement = 0.01 m; that is, this measurement of length has been made to the nearest 0.01 of a metre. Limit of reading = 0.01 m	Find the limit of reading (0.01 m) and absolute error (0.005 m) as in Worked Example 2. Lower limit = 9.38 m − 0.005 m Upper limit = 9.38 m + 0.005 m	*Note:* The true measurement is greater than or equal to the lower limit, but is less than the upper limit; that is, lower limit ⩽ true measurement < upper limit.
	ii	Absolute error = $\frac{1}{2} \times 0.01$ m = 0.005 m		
	iii	Lower limit = 9.38 − 0.005 = 9.375 m Upper limit = 9.38 + 0.005 = 9.385 m True measurement is between 9.375 and 9.385 m.		

8 Complete the following for a measurement of 2.6 kg.
  a Limit of reading = _____ kg   **b** Absolute error = _____ kg
  c Lower limit of measurement = 2.6 − _____ kg. Upper limit of measurement = 2.6 + _____ kg
     The true measurement lies between _____ and _____.

9 For each of the measurements below, find:
    i the limit of reading   **ii** the absolute error in the measurement
   iii the lower and upper limits of the true measurement.

  a 12 mm   **b** 348 g   **c** 375 mL   **d** 8.2 km
  e 18.4 s   **f** 4.9 kg   **g** 2.37 m   **h** 5.81 L

MEASUREMENT

WORKED EXAMPLE 4

The mass of a car was given as 2300 kg to the nearest 100 kg. Find:

a the limit of reading   **b** the greatest possible error (GPE) in the measurement

c the lower and upper limits of the true measurement.

	Solve	Think	Apply
a	The smallest unit used for this measurement is given as 100 kg. Limit of reading $=$ 100 kg	Find the limit of reading (100 kg) and GPE (50 kg) as in Worked Example 2. Lower limit $=$ 2300 kg $-$ 50 kg Upper limit $=$ 2300 kg $+$ 50 kg	Find the limit of reading and the greatest possible error. Lower limit $=$ measurement $-$ GPE Upper limit $=$ measurement $+$ GPE
b	GPE $= \frac{1}{2} \times$ 100 kg$=$ 50 kg		
c	Lower limit $=$ 2300 $-$ 50 $=$ 2250 kg Upper limit $=$ 2300 $+$ 50 $=$ 2350 kg The true measurement lies between 2250 kg and 2350 kg.		

10 The capacity of a container is given as 750 mL to the nearest 50 mL. Complete the following.

    a The measurement has been given to the nearest _____ mL. Limit of reading $=$ _____ mL.

    b GPE $= \frac{1}{2} \times$ _____ mL $=$ _____ mL.

    c Lower limit of measurement $=$ 750 $-$ _____ mL.

        Upper limit of measurement $=$ 750 $+$ _____ mL.

        The true measurement lies between _____ and _____.

11 For each of the measurements below, find:

     i the smallest unit of measurement

    ii the greatest possible error (GPE) in the measurement

   iii the lower and upper limits of the true measurement.

    a The mass of a can of soup is 420 g, to the nearest 30 g.

    b The capacity of a drink bottle is 380 mL, to the nearest 20 mL.

    c The crowd at a cricket match was 38 000, to the nearest 1000.

    d The time taken for a plane flight was $6\frac{1}{2}$ hours, to the nearest $\frac{1}{2}$ hour.

WORKED EXAMPLE 5

a Find the GPE for the measurement 18 kg.

b Express the GPE as a percentage of the measurement.

Solve	Think	Apply
a Smallest unit of measurement $= 1$ kg GPE $= 0.5$ kg	Find the limit of reading (1 kg) and GPE (0.5 kg) as in Worked Example 2.	The percentage error is the GPE expressed as a percentage of the given measurement.
b Percentage error $= \pm\dfrac{0.5}{18} \times 100\%$ $= \pm2.8\%$ (1 decimal point)	Percentage error $= \pm\dfrac{0.5}{18} \times 100\%$	Percentage error $= \pm\dfrac{\text{GPE}}{\text{measurement}} \times 100\%$

12 Complete the following.

For a measurement of 7.6 m, the limit of reading = _____ m.

GPE = _____ m

Percentage error $= \pm\dfrac{\square}{7.6} \times 100\% = \pm$____% (to 1 decimal place).

13 For each of the measurements below, find:

    i the greatest possible error   **ii** the percentage error for each.

a 10 cm   **b** 32 s   **c** 250 g   **d** 14 min

e 6 L   **f** 2.4 kg   **g** 13.5 s   **h** 12.56 m

WORKED EXAMPLE 6

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.

a Calculate the perimeter of the rectangle using these measurements.

b Find the lower and upper limits of its true perimeter.

c Hence find the maximum error in the answer to part **a**.

Solve	Think	Apply
a Perimeter $= 2 \times 8 + 2 \times 6$ $= 28$ cm	Perimeter (using measurements) $= 2 \times l + 2 \times b = 28$ cm	Calculate the perimeter using the measured length and breadth.
b Now 7.5 cm $\leqslant$ length < 8.5 cm and 5.5 cm $\leqslant$ breadth < 6.5 cm. Hence, $2 \times 7.5 + 2 \times 5.5$ cm $\leqslant$ perimeter $< 2 \times 8.5 + 2 \times 6.5$ cm. Thus 26 cm $\leqslant$ perimeter < 30 cm.	The GPE of each measurement is 0.5 cm: length lies between 7.5 cm and 8.5 cm and breadth between 5.5 cm and 6.5 cm. Lower limit of perimeter $= 2 \times 7.5 + 2 \times 5.5 = 26$ cm Upper limit of perimeter $= 2 \times 8.5 + 2 \times 6.5 = 30$ cm	Determine the lower and upper limits of each given measurement. Calculate the perimeter using the lower and upper limits of length and breadth.
c Maximum error $= 28$ cm $- 26$ cm (or 28 cm $- 30$ cm) $= \pm2$ cm	Maximum error $=$ perimeter (using given measurements) $-$ lower limit of perimeter (or upper limit of perimeter).	Find the difference between the perimeter, calculated using the given measurements, and the perimeter using the lower (or upper) limit of each.

14 The length and breadth of a rectangular recreation room are measured to be 7 m and 4 m, respectively. Complete the following.

a Using the given measurements,

Perimeter $= 2 \times$ ____ $+ 2 \times$ ____

$\qquad = $ ____ m

b Now 6.5 m $\leqslant$ length $<$ ____ m

and ____ m $\leqslant$ breadth $<$ 4.5 m

Lower limit of perimeter

$= 2 \times 6.5 + 2 \times 3.5$ m $=$ ____ m

Upper limit of perimeter

$= 2 \times$ ____ $+ 2 \times$ ____ m $=$ ____ m

Thus ____ m $\leqslant$ perimeter $<$ ____ m
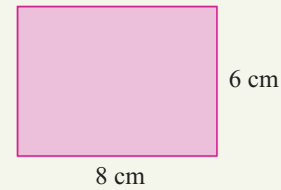
c Maximum error $=$ ____ m $-$ ____ m

$\qquad = \pm$ ____ m

15 The length and breadth of a rectangle were measured to be 9 cm and 5 cm respectively.

a Calculate the perimeter of the rectangle using these measurements.

b Find the lower and upper limits of its true length and breadth.

c Hence, find the lower and upper limits of its true perimeter.

d Find the maximum error in the answer to part **a**.

16 Two pieces of timber were measured to be 164 cm and 128 cm respectively.

a If the two pieces were placed end to end, what would be their total length, using the measurements given?

b Find the lower and upper limits of the true length of each piece.

c Hence, calculate the lower and upper limits of the true total length of these two pieces of timber.

d Find the maximum error in the answer to part **a**.

17 The masses of two bags of sand were measured and found to be 47 kg and 52 kg.

a What is the total mass of the two bags?

b Find the lower and upper limits of the true mass of each bag.

c Hence, calculate the lower and upper limits of the true total mass.

d What is the maximum error in the answer to part **a**?

18 Repeat question **17** given that the masses of the sand bags were 47.4 kg and 51.9 kg.

WORKED EXAMPLE 7

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.
a Calculate the area using the measurements given.
b Find the lower and upper limits of the true area.
**c** Hence, find the maximum error in the answer to part **a**.

	Solve	Think	Apply
a	Area $= 8 \times 6 = 48$ cm²	Area (using given measurements) $= l \times b = 48$ cm²	Calculate the area using the measured length and breadth.
b	Now 7.5 cm $\leqslant$ length $<$ 8.5 cm and 5.5 cm $\leqslant$ breadth $<$ 6.5 cm. Hence, 7.5×5.5 cm² $\leqslant$ area $< 8.5 \times 6.5$ cm². Thus 41.25 cm² $\leqslant$ area $<$ 55.25 cm².	The GPE of each measurement is 0.5 cm. Hence, the length lies between 7.5 cm and 8.5 cm and the breadth lies between 5.5 cm and 6.5 cm. Lower limit of area $= 7.5 \times 5.5 = 41.25$ cm² Upper limit of area $= 8.5 \times 6.5 = 55.25$ cm²	Determine the lower and upper limits of each given measurement. Calculate the area using the lower limits of the length and breadth, and calculate the area using the upper limits of the length and breadth. Find the difference between the area calculated using the given measurements and the area using the lower (or upper) limit of each measurement.
c	$48 - 41.25 = 6.75$ cm² $48 - 55.25 = -7.25$ cm² Maximum error $= 7.25$ cm²	Maximum error = area (using given measurements) − lower limit of area (or upper limit of area).	

19 The length and breadth of a rectangle are measured to be 7 cm and 4 cm respectively. Complete the following.
  a Using the given measurements, area = _____ × _____ cm² = _____ cm²
  b Now 6.5 cm $\leqslant$ length $<$ _____ cm and _____ cm $\leqslant$ breadth $<$ 4.5 cm
    Hence, lower limit of area $= 6.5 \times 3.5$ cm² = _____ cm²
        upper limit of area = _____ × _____ cm² = _____ cm²
    Thus _____ cm² $\leqslant$ area $<$ _____ cm²
  c _____ − lower limit of area = _____ cm²
    _____ − upper limit of area = _____ cm²
    ∴ Maximum error $= \pm$ _____ cm²

20 A rectangular room was measured to be 5 m long by 3 m wide.
  a Calculate the area of the room using these measurements.
  b Find the lower and upper limits of the true length and width.
  c What are the lower and upper limits of the true area?
  d Find the maximum error in the answer to part **a**.
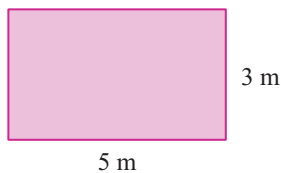
21 Repeat question **20** for a room 5.4 m long by 3.2 m wide.

22 The diameter of a circular pizza tray is measured to be 28.6 cm.

   a Calculate the area of the tray using the measurement given. (Remember $A = \dfrac{\pi d^2}{4}$.)

   b What are the lower and upper limits of the true length of the diameter?

   c Find the lower and upper limits of the true area of the tray.

   d What is the maximum error in the answer to part **a**?

2F Accuracy of measurement

As a result of the accumulating effect of errors when calculations are performed with measured values, the following conventions are usually applied.

- When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement with the least decimal place accuracy.
- When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

WORKED EXAMPLE 1

Give a sensible approximation for the result of this calculation: 15.642 m + 8 m + 19.21 m.

Solve	Think	Apply
15.642 m + 8 m + 19.21 m = 42.852 m = 43 m to the nearest metre	15.642 m is accurate to 3 decimal places. 8 m is accurate to the nearest whole number. 19.21 m is accurate to 2 decimal places. Least precise measurement is 8 m (the nearest whole number), hence the answer should be rounded to the nearest whole number.	When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement with the least decimal place accuracy.

EXERCISE 2F

1 Complete the following to calculate 13.65 L + 10.9 L + 12.624 L.

    13.65 is accurate to _____ decimal place(s).

    10.9 is accurate to _____ decimal place(s).

    12.624 is accurate to _____ decimal place(s).

    Least precise measurement is _____ L to _____ decimal place(s).

    Hence, 13.65 L + 10.9 L + 12.624 L = _____ L

                               = _____ L to _____ decimal place(s)

SAMPLE

2 Give a sensible approximation for the results of the following calculations.

   a 9.87 m + 15.219 m + 11 m   **b** 27.3 L + 21.475 L + 16.54 L

   c 6.132 km − 3.46 km   **d** 10.528 kg + 11.607 kg − 9.2 kg

WORKED EXAMPLE 2

Give a sensible approximation for the result of this calculation: 15.2 m × 9.8 m.

Solve	Think	Apply
15.2 m × 9.8 m = 148.96 m² $\quad$ = 150 m² to 2 significant figures	15.2 is accurate to 3 significant figures. 9.8 is accurate to 2 significant figures. The measurement with the least number of significant figures is 9.8 m (2 significant figures), hence the answer should be rounded to 2 significant figures.	When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

3 Complete the following to calculate 7.5 m × 12.3 m.

   7.5 m is accurate to ____ significant figures.

   12.3 m is accurate to ____ significant figures.

   The least precise measurement is ____ m (to ____ significant figures).

   Hence, 7.5 m × 12.3 m = ____ m²

                   = ____ m² (to ____ significant figures)

4 Give a sensible approximation for the result of the following calculations.

   a 23.6 m × 5.7 m   **b** 405.2 cm × 58.6 cm   **c** 88 cm³ ÷ 65 cm³   **d** 37.7 mm ÷ 12 mm

2G   Rates

A **rate** is a comparison between quantities of different kinds.

The comparison is made by dividing one quantity by the other, in the required order.

The rate is then expressed in the form 'the first quantity per unit of the second quantity'.

We may compare distance travelled with petrol used, amount of fertiliser needed with area of land, amount of pay with time worked, etc.

WORKED EXAMPLE 1

Wendy types 600 words in 8 min. How many words per minute does she type?

Solve	Think	Apply
Rate = $\dfrac{600 \text{ words}}{8 \text{ min}}$ = 75 words/min	This rate is comparing number of words with time, in that order. Thus divide the number of words by the time. Rate = 600 words in 8 min $\quad$ = 600 ÷ 8 = 75 words/min This is the number of words per unit of time.	Compare the quantities by dividing one quantity by the other.

MEASUREMENT

WORKED EXAMPLE 2

A car used 49 L of petrol on a trip of 500 km.
a Calculate the number of kilometres the car travels per litre of petrol consumed.
b Calculate the number of litres of petrol the car consumes per kilometre of travel.

	Solve	Think	Apply
a	$\text{Rate} = \dfrac{500 \text{ km}}{49 \text{ L}}$ $= 10.2 \text{ km/L}$ (1 decimal place)	This rate is comparing the distance travelled with the amount of petrol consumed, in that order. Rate = 500 km on 49 L $= 500 \div 49 = 10.2 \text{ km/L}$ (1 decimal place) Car travels 10.2 km for every 1 L of petrol used.	Determine the order in which the two quantities are being compared and divide one by the other, in that order.
b	$\text{Rate} = \dfrac{49 \text{ L}}{500 \text{ km}}$ $= 0.098 \text{ L/km}$	Rate = 49 L for 500 km $= 49 \div 500 = 0.098 \text{ L/km}$ This is the amount of petrol consumed per unit of distance travelled: car used 0.098 L of petrol for every kilometre. As this rate is often small, petrol consumption is usually quoted as the amount of petrol used per 100 km. Rate = 0.098 L/km $= 0.098 \times 100 \text{ L/100 km}$ $= 9.8 \text{ L/100 km}$	Remember to write the units.

EXERCISE 2G

1 a A 2.5 kg box of soap powder costs $10.90. Complete the following to find the cost per kg.

$\text{Rate} = \dfrac{\text{cost}}{\text{weight}} = \dfrac{\$\Box}{\Box \text{ kg}}$

$= \underline{} \text{ \$/kg or \$}\underline{}\text{/kg}$

b Jenny typed 300 words in 5 min. Complete the following to find her typing rate per minute.

$\text{Rate} = \dfrac{\text{number of words}}{\text{time}} = \dfrac{\Box \text{ words}}{\Box \text{ min}}$

$= \underline{} \text{ words/min}$

c A truck used 114 L of petrol on a trip of 600 km. Complete the following to express petrol consumption in L/km.

$\text{Rate} = \dfrac{\text{amount of petrol used}}{\text{distance travelled}} = \dfrac{\Box \text{ L}}{\Box \text{ km}}$

$= \underline{} \text{ L/km}$

d Vicki was paid $82.50 for 6 h work. To calculate her rate of pay, complete the following.

$\text{Rate} = \dfrac{\text{amount earned}}{\text{hours worked}} = \dfrac{\$\Box}{\Box \text{ h}}$

$= \underline{} \text{ \$/h or \$}\underline{}\text{/h}$

e Howard had to pay $55.20 for 120 telephone calls. To calculate the cost per call, complete the following.

$\text{Rate} = \dfrac{\text{total cost}}{\text{number of calls}} = \dfrac{\$\Box}{\Box \text{ calls}}$

$= \underline{} \text{ \$/call or \$}\underline{}\text{/call}$

2 a The temperature rose 14°C in $3\frac{1}{2}$ h. At what rate, in degrees per hour, did the temperature rise?

b Jeremy spread 24 kg of fertiliser over an area of 60 m². Calculate the rate of application in kg per m².

c Fred's electricity bill was \$174.72 for 1560 kilowatt hours of power (kWh). What was the cost of electricity per kWh?

d Peggy drove 195 km in $2\frac{1}{2}$ h at a constant speed. Calculate her speed in km/h.

e Calculate the flow rate per minute if 119 L of water flows through a pipe in 35 min.

WORKED EXAMPLE 3

Convert 8 t/ha to:

a kg/ha **b** kg/m² **c** g/m²

	Solve	Think	Apply
a	8 t/ha = 8000 kg/ha	Change 8 t to 8000 kg.	Convert the given units to the required units and divide in the order given.
b	$8 \text{ t/ha} = \dfrac{8000 \text{ kg}}{10\,000 \text{ m}^2}$ $= 8000 \div 10\,000 \text{ kg/m}^2$ $= 0.8 \text{ kg/m}^2$	Change 8 t to 8000 kg and 1 ha to 10 000 m² and divide.	
c	$8 \text{ t/ha} = \dfrac{8 \times 1000 \times 1000 \text{ g}}{10\,000 \text{ m}^2}$ $= 8\,000\,000 \div 10\,000 \text{ g/m}^2$ $= 800 \text{ g/m}^2$	Change 8 t to $8 \times 1000 \times 1000$ g and 1 ha to 10 000 m² and divide.	

WORKED EXAMPLE 4

Convert 72 L/h to mL/s.

Solve	Think	Apply
$72 \text{ L/h} = \dfrac{72\,000 \text{ mL}}{(60 \times 60) \text{ s}}$ $= 72\,000 \div (60 \times 60) \text{ mL/s}$ $= 20 \text{ mL/s}$	Change 72 L to 72 000 mL and 1 h to 60×60 s and divide.	Convert the given units to the required units and divide in the order given.

3 Complete each of the following to convert:

a 16 t/ha to kg/m²

$\text{Rate} = \dfrac{16 \text{ t}}{1 \text{ ha}} = \dfrac{\square \text{ kg}}{\square \text{ m}^2} = \underline{\quad\quad} \text{ kg/m}^2$

b \$3.75/h to cents/min

$\text{Rate} = \dfrac{\$3.75}{1 \text{ h}} = \dfrac{\square \text{ cents}}{\square \text{ min}} = \underline{\quad\quad} \text{ c/min}$

c \$1.80/m to cents/mm

$\text{Rate} = \dfrac{\$1.80}{1 \text{ m}} = \dfrac{\square \text{ cents}}{\square \text{ mm}} = \underline{\quad\quad} \text{ c/mm}$

d 15%/year to %/month

$\text{Rate} = \dfrac{15\%}{1 \text{ year}} = \dfrac{15\%}{\square \text{ months}} = \underline{\quad\quad} \%\text{/month}$

e 2.4 kg/L to g/mL

$\text{Rate} = \dfrac{2.4 \text{ kg}}{1 \text{ L}} = \dfrac{\square \text{ g}}{\square \text{ mL}} = \underline{\quad\quad} \text{ g/mL}$

4 Convert the following.

a 45 L/h to mL/s **b** \$12/kg to cents/g **c** 18 km/h to m/s

d 27 L/h to mL/s **e** 7.2 kg/day to g/min **f** 14 km/h to cm/s

WORKED EXAMPLE 5

Convert 5 m/s to km/h.

Solve	Think	Apply
$5 \text{ m/s} = \dfrac{5 \times 60 \times 60}{1000} \text{ km/h}$ $= 18 \text{ km/h}$	Multiply the number of metres travelled in 1 s by the number of seconds in 1 h. $5 \text{ m/s} = 5 \times (60 \times 60) \text{ m/h}$ $= 18\,000 \text{ m/h}$ $= 18\,000 \div 1000 \text{ km/h}$ Dividing by the number of m in a km. $= 18 \text{ km/h}$	Multiply by the number of seconds in an hour and divide by the number of metres in a kilometre.

5 a To convert 6 m/s to km/h complete the following.

The number of seconds in 1 hour = _____ × _____

The number of metres in a kilometre = _____

Hence, 6 m/s = $\dfrac{6 \times \square \times \square}{\square}$ km/h = _____ km/h

b To convert 3 cents/minute to $/day, complete the following.

The number of minutes in 1 day = 24 × _____

The number of cents in $1 = _____

Hence, 3 cents/min = $\dfrac{3 \times 24 \times \square}{\square}$ $/day

= _____ $/day or $_____ /day

6 a Convert 5 g/mL to: **i** g/L   **ii** kg/L

b Convert 0.8c/g to: **i** c/kg   **ii** $/kg

c Convert 0.75 kg/m² to: **i** kg/ha   **ii** t/ha

d Convert 0.4 mL/s to: **i** mL/h   **ii** L/h

e Convert 0.8c/m to: **i** c/km   **ii** $/km

7 A car travels at 60 km/h.

a How far will it travel in $2\frac{1}{2}$ hours?   **b** How long will it take to travel 225 km?

8 Fertiliser is to be spread at the rate of 0.2 kg/m².

a How much fertiliser would be needed for an area of 600 m²?

b If the fertiliser is sold in 50 kg bags, how many bags are needed for an area of 600 m²?

c What area could be fertilised with 1 t of fertiliser?

9 A patient in hospital is given an antibiotic solution intravenously at the rate of 80 mL/h.
  a How much antibiotic solution will the patient receive in $6\frac{1}{2}$ h?
  b How often would 600 mL containers of antibiotic solution need to be changed?
  c If 1 mL of this solution contains 15 drops, calculate the rate at which the patient receives the antibiotic in drops/minute.

10 Anna is paid at the rate of $23.60/h.
  a How much would she be paid for working 15 h?
  b How long would she need to work to earn $472?
  c Anna wants to save for a trip. If she works 28 h in week 1, and 31 h in the next week, how many more hours does she need to work to earn $2000?

11 The conversion rate for Australian dollars (A$) into American dollars (US$) is 0.95 A$/US$, (US$1 = A$0.95).
  a How many US dollars would I receive for A$2750?
  b How many Australian dollars would I receive for US$1300?
  c Convert US$1800 into A$.

12 On a trip of 400 km a car uses 30 L of petrol.
  a Express the fuel consumption in L/100 km.
  b Assuming the same rate of fuel consumption:
    i how much fuel would the car use for a trip of 500 km?
    ii how far could the car travel on a full tank of 45 L?

13 The average distance of the Earth from the Sun is 1.49×10^8 km. Assume the Earth travels in a circular orbit around the Sun.
  a Calculate (to 3 significant figures) the distance travelled by the Earth in one complete orbit. ($C = 2\pi r$)
  b It takes the Earth 1 year (365.25 days) to travel this distance. Using the answer from part **a**, find the average speed at which the Earth travels through space in:
    i km/h   ii km/s

14 If it costs 15 cents for 1 kilowatt (1000 W) of power for 1 h, how much does it cost to run a 2400 W heater from 5 pm to 11 pm?

2H Ratios

A **ratio** is a comparison between quantities of the same kind.
A ratio can be written using colon notation or as a fraction.
A ratio does not have units.

Ratios are simplified by multiplying or dividing each term of the ratio by the same number.

MEASUREMENT

WORKED EXAMPLE 1

Tom's height was 169 cm and Laura's height was 165 cm. Write the ratio of:

a Tom's height to Laura's height
b Laura's height to Tom's height.

Solve	Think	Apply
a Tom : Laura = 169 : 165 or $\frac{169}{165}$	The quantities compared are 169 cm and 165 cm. Write these quantities in the order given, separated by a colon or as a fraction, leaving out the units.	Write the quantities in the order given, separated by a colon or as a fraction, leaving out the units. The order is important as 169 : 165 ≠ 165 : 169.
b Laura : Tom = 165 : 169 or $\frac{165}{169}$		

WORKED EXAMPLE 2

Simplify these ratios.

a 24 : 18   **b** 16 : 12 : 20   **c** 1.5 : 2.7   **d** $1\frac{3}{4} : \frac{2}{3}$ **e** 85 cm : 1.2 m

Solve	Think	Apply
a 24 : 18 = 4 : 3	24 : 18 = 4 : 3 (dividing each term by 6) *Calculator:* Press: 24 $a\frac{b}{c}$ 18 = SHIFT d/c Display is 4	3	Ratios are simplified by multiplying or dividing each term by the same number.
b 16 : 12 : 20 = 4 : 3 : 5	16 : 12 : 20 = 4 : 3 : 5 (dividing each term by 4)	
c 1.5 : 2.7 = 15 : 27 = 5 : 9	1.5 : 2.7 = 15 : 27 (multiplying both terms by 10) = 5 : 9 (dividing each term by 3)	
d $1\frac{3}{4} : \frac{2}{3} = \frac{7}{4} \times 12 : \frac{2}{3} \times 12$ = 21 : 8	$1\frac{3}{4} : \frac{2}{3} = \frac{7}{4} \times 12 : \frac{2}{3} \times 12$ (multiplying each term by the lowest common denominator: 12) = 21 : 8	
e 85 cm : 1.2 m = 85 : 120 = 17 : 24	85 cm : 1.2 m = 85 cm : 120 cm (same units) = 17 : 24 (dividing each term by 5)	

EXERCISE 2H

1 The table shows the number of each type of vehicle that passes the front of a school in an hour.

Write the ratio of the number of:

a cars to trucks
b trucks to cars
c motorcycles to cars
d trucks to buses
e cars to trucks to motorcycles
f trucks to motorcycles to buses.

Type of vehicle	Number
Car	54
Truck	13
Motorcycle	7
Bus	4
Other	2

2 Simplify these ratios.

a 25 : 35 **b** 27 : 18 **c** 84 : 48 **d** 12 : 24 : 18

e 36 : 48 : 72 **f** 1.6 : 1.9 **g** 1.6 : 1.8 **h** 0.56 : 0.32

i 0.93 : 0.6 **j** 0.256 : 0.8 **k** $\frac{1}{2} : \frac{1}{3}$ **l** $1\frac{1}{2} : 2\frac{1}{2}$

m $\frac{5}{8} : \frac{2}{3}$ **n** 25 cm : 1.1 m **o** 2.2 kg : 850 g **p** \$1.50 : 80 cents

q $1\frac{1}{2}$ h : 40 min **r** 6 min : 2.1 h **s** 600 mL : 1.5 L : 2.1 L **t** 40 g/L : 3 mg/mL : 2 mg/mL

WORKED EXAMPLE 3

Express the following ratios in the form $n : 1$.

a 25 : 10 **b** 16 : 24

	Solve	Think	Apply
a	$25 : 10 = \frac{25}{10} : 1$ $= 2.5 : 1$	Divide both numbers by 10. $25 : 10 = \frac{25}{10} : \frac{10}{10} = 2.5 : 1$ This means that the first number is 2.5 times the second.	To put the ratio in the form $n : 1$, divide the first term by the second.
b	$16 : 24 = \frac{16}{24} : 1$ $= \frac{2}{3} : 1$ or $0.\dot{6} : 1$	Divide both numbers by 24. $16 : 24 = \frac{16}{24} : \frac{24}{24} = \frac{2}{3} : 1$ or $0.\dot{6} : 1$ This means that the first number is $\frac{2}{3}$ (or $0.\dot{6}$.) times the second.	

3 Express these ratios in the form $n : 1$.

a 35 : 10 **b** 72 : 40 **c** 24 : 60 **d** 72 : 80

4 Simplify and express the following ratios in the form $n : 1$. Explain the meaning of the answer.

a 3.5 m : 70 cm **b** 1.2 kg : 800 g **c** $2\frac{1}{2}$ cups : $\frac{1}{2}$ cup **d** 0.04 ha : 500 m²

5 Express these ratios in the form $1 : n$.

a 50 : 87 **b** 40 : 90 **c** 60 : 48 **d** 125 : 8

e 200 : 154 **f** 1 cm : 1 m **g** 1 mm : 1 m **h** 20 mL : 1 L

6

Category	Number who develop lung cancer	Ratio (1 : n)
Males who have never smoked	10 in 760	1 : 76
Males who have smoked	25 in 300	
Males who currently smoke	20 in 90	
Females who have never smoked	3 in 471	
Females who have smoked	50 in 1150	
Females who currently smoke	15 in 132	

a Complete the table to express the risk, in the form $1 : n$, of developing lung cancer.

b Which group of people is most likely to develop lung cancer?

c Which group of people is least likely to develop lung cancer?

WORKED EXAMPLE 4

The ratio of the number of boys to girls in a school is 7 : 6.
If there are 354 girls in the school, how many boys are there?

Solve	Think	Apply
$\dfrac{\text{boys}}{354} = \dfrac{7}{6}$ $\therefore \text{boys} = \dfrac{7}{6} \times 354$ $= 413$ There are 413 boys.	boys : girls = 7 : 6 boys : 354 = 7 : 6 Using fraction notation $\dfrac{\text{boys}}{354} = \dfrac{7}{6}$ (Remember the order of terms is important!) Treat this as an equation. Solve by multiplying both sides by 354. $\dfrac{\text{boys}}{354} \times 354 = \dfrac{7}{6} \times 354$ $\therefore \text{boys} = \dfrac{7}{6} \times 354 = 413$	Using fraction notation, write two equivalent ratios. Treat this as an equation and solve.

7 The ratio of the number of boys to girls in a school is 9 : 8 and there are 312 girls at the school. Complete the following to find the number of boys.

$$\frac{\text{boys}}{\square} = \frac{9}{8}$$

$\therefore \text{boys} = \frac{9}{8} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

8 At an electrical store the ratio of profit to sales is 2 : 7. If the annual sales for the year were $145 600, what was the annual profit?

9 The ratio of Ben's net salary to the tax he pays is 10 : 3. Find his net salary if he paid $14 580 in tax for 1 year.

WORKED EXAMPLE 5

The ratio of boys to girls in a school is 8 : 7.
If there are 264 boys in the school, how many girls are there?

Solve	Think	Apply
$\dfrac{264}{\text{girls}} = \dfrac{8}{7}$ $\dfrac{\text{girls}}{264} = \dfrac{7}{8}$ $\therefore \text{girls} = \dfrac{7}{8} \times 264$ $= 231$ There are 231 girls.	boys : girls = 8 : 7 girls : boys = 7 : 8 Using fraction notation $\dfrac{\text{girls}}{264} = \dfrac{7}{8}$ Treat this as an equation and solve by multiplying both sides by 264. $\dfrac{\text{girls}}{264} \times 264 = \dfrac{7}{8} \times 264$ $\therefore \text{girls} = \dfrac{7}{8} \times 264 = 231$	Using fraction notation, write two equivalent ratios. Change the order so that the unknown is the first term of the ratio. Treat this as an equation and solve.

10 The ratio of males to females at a basketball match is 10 : 9. If there are 470 males at the match, complete the following to find the number of females.

$$\frac{\square}{\text{females}} = \frac{10}{9} \qquad \text{so} \qquad \frac{\text{females}}{\square} = \frac{9}{10}$$

$\therefore \text{females} = \frac{9}{10} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

11 The ratio of a daughter's height to that of her mother is 4 : 5. What is the mother's height if her daughter is 172 cm tall?

12 A farmer plants lemon trees and orange trees in an orchard in the ratio 2 : 5.
   a In one orchard he planted 60 orange trees. How many lemon trees did he plant?
   b In another orchard he planted 18 lemon trees. How many orange trees did he plant?

WORKED EXAMPLE 6

An inheritance of \$24 000 is to be divided between Sam and Jamie in the ratio 2 : 3.
How much will each receive?

Solve	Think	Apply
The money needs to be divided into $2 + 3 = 5$ parts. Sam receives $\frac{2}{5} \times \$24\ 000 = \$9600$ Jamie receives $\frac{3}{5} \times \$24\ 000 = \$14\ 400$	For every \$2 that Sam receives, Jamie receives \$3. Thus Sam receives \$2 out of every \$5, or $\frac{2}{5}$ of the inheritance, and Jamie receives \$3 out of every \$5, or $\frac{3}{5}$ of the inheritance.	Add the terms of the ratio. This is the number of parts into which the quantity is to be divided. The relevant fraction of the quantity can then be found.

13 If \$30 000 is to be divided between Hannah and Rachel in the ratio 5 : 3, how much will each girl receive?

14 If \$1500 is divided in the ratio 3 : 1, how much is the larger share?

15 Cordial and water are mixed in the ratio 1 : 8 to make a fruit drink. How much cordial and water would there be in a 180 mL glass of fruit drink?

16 A metal solder is made by combining lead and tin in the ratio 2 : 3. How much tin is needed to make 10 kg of the metal solder?

17 The masses of Alison, Vincent and Matthew are in the ratio 4 : 3 : 5. If their combined mass is 168 kg, find the mass of each person.

18 An investment fund has investments in property, shares and government bonds in the ratio 5 : 3 : 2.
   a If the fund has a total of \$1.8 million invested, find the amount invested in each of these three areas.
   b If during the next year the fund manager decides to transfer \$160 000 from investments in government bonds to shares, find the new ratio of investments in this fund.

2I Percentage change

WORKED EXAMPLE 1

a Increase 180 by 14%.   **b** Decrease 96 by 35%.

	Solve	Think	Apply
a	114% of $180 = \dfrac{114}{100} \times 180$ $= 205.2$	$180 + 14\%$ of 180 $= 100\%$ of $180 + 14\%$ of 180 $= 114\%$ of 180 $= \dfrac{114}{100} \times 180$ $= 205.2$	To increase a quantity by $x\%$, find $(100 + x)\%$ of the quantity.
b	65% of $96 = \dfrac{65}{100} \times 96$ $= 62.4$	$96 - 35\%$ of 96 $= 100\%$ of $96 - 35\%$ of 96 $= 65\%$ of 96 $= \dfrac{65}{100} \times 96$ $= 62.4$	To decrease a quantity by $x\%$, find $(100 - x)\%$ of the quantity.

EXERCISE 2I

13D

1 Increase the following.
- **a** 160 by 12%
- **b** 240 by 35%
- **c** 850 by 8%
- **d** 466 by 20%
- **e** 285 by 100%
- **f** 300 by $7\frac{1}{4}\%$

2 Decrease the following.
- **a** 86 by 25%
- **b** 350 by 40%
- **c** 780 by 16%
- **d** 115 by 30%
- **e** 480 by 24%
- **f** 520 by $12\frac{1}{2}\%$

WORKED EXAMPLE 2

a Increase $75 by 20%.   **b** Decrease 3 m by 12%.

	Solve	Think	Apply
a	120% of $\$75 = \dfrac{120}{100} \times \$75$ $= \$90$	$\$75 + 20\%$ of $\$75$ $= 100\%$ of $\$75 + 20\%$ of $\$75$ $= 120\%$ of $\$75$ $= \dfrac{120}{100} \times \$75 = \$90$	To increase a quantity by $x\%$, find $(100 + x)\%$ of the quantity.
b	88% of $3 \text{ m} = \dfrac{88}{100} \times 3 \text{ m}$ $= 2.64 \text{ m}$	$3 \text{ m} - 12\%$ of 3 m $= 100\%$ of $3 \text{ m} - 12\%$ of 3 m $= 88\%$ of 3 m $= \dfrac{88}{100} \times 3 \text{ m} = 2.64 \text{ m}$	To decrease a quantity by $x\%$, find $(100 - x)\%$ of the quantity.

MEASUREMENT

3 Increase the following.

   a $450 by 28%   **b** 15 m by 75%   **c** 2 t by 1.5%

   d $6 by 200%   **e** 40 s by $62\frac{1}{2}$%   **f** 300 L by $6\frac{2}{3}$%

4 Decrease the following.

   a 4.8 km by 19%   **b** 120 kg by 13.2%   **c** 57 s by $33\frac{1}{3}$%

   d $456 by 8.5%   **e** $3000 by $2\frac{1}{4}$%   **f** 4.2 ha by 15.6%

5 What would be the value of a $10 000 share portfolio at the end of 2 years if the shares:

   a increase in value by 15% in the first year and then increase in value by 12% in the second year?

   b increase in value by 20% in the first year and then decrease in value by 8% in the second year?

   c decrease in value by 14% in the first year and then increase in value by 16% in the second year?

   d increase in value by 10% in the first year and then decrease in value by 10% in the second year?

   e decrease in value by 10% in the first year and then increase in value by 10% in the second year?

   f decrease in value by 25% in the first year and then increase in value by 33% in the second year?

6 A car that cost $28 900 new depreciated in value by 22% in the first year, by 20% in the second year and by 18% in the third year. What was its value at the end of 3 years?

$$\text{Percentage change} = \frac{\text{change in value}}{\text{original value}} \times 100\%$$

WORKED EXAMPLE 3

A stamp collection was bought for $3600. In the first year its value increased by 5%. In the second year it increased in value by a further 6%.

a Calculate its value at the end of the first year.

b Calculate its value at the end of the second year.

c What is the overall change in its value after 2 years?

d Find the percentage change in value over the 2 years.

	Solve	Think	Apply
a	Value after 1 year = 105% of $3600 $= \frac{105}{100} \times 3600 = \$3780$	Calculate the value at the end of the first year. Use this result to calculate the value at the end of the second year. Find the overall change in value over these 2 years. Calculate the percentage change in value over these 2 years.	Percentage change $= \frac{\text{change in value}}{\text{original value}} \times 100\%$
b	Value after 2 years = 106% of $3780 $= \frac{106}{100} \times 3780 = \$4006.80$		
c	Overall change in value = $4006.80 − $3600 = $406.80		
d	Percentage change in value over 2 years $= \frac{\text{change in value}}{\text{original value}} \times 100\%$ $= \frac{406.8}{3600} \times 100\% = 11.3\%$		

MEASUREMENT

7 A painting was bought for \$3600. In the first year its value increased by 6%. In the second year it increased by a further 8%. Complete the following.

a Value after 1 year = 106% of \$____

$$= \frac{\square}{100} \times \$\underline{}$$

$$= \$\underline{}$$

b Value after 2 years = ____% of \$____

$$= \frac{\square}{100} \times \$\underline{}$$

$$= \$\underline{}$$

c Overall change in value after 2 years = \$____ − \$3600

$$= \$\underline{}$$

d Percentage change in value over the 2 years $= \frac{\square}{\square} \times 100\%$

$$= \underline{}\% \text{ (to 1 decimal place)}$$

8 A \$50 000 share portfolio increased in value by 12% in the first year and decreased by 3% in the second year. Complete the following to find the overall percentage change in value of the portfolio after 2 years.

Value after 1 year $= \frac{112}{100} \times \$50\,000 = \$\underline{}$

Value after 2 years $= \frac{97}{100} \times \$\underline{} = \$\underline{}$

Overall change in value over 2 years $= \$\underline{} - \$50\,000$

$$= \$\underline{}$$

Percentage change in value over 2 years $= \frac{\square}{50\,000} \times 100\%$

$$= \underline{}\%$$

9 An investor bought \$80 000 worth of gold. In the first year the gold increased in value by 10%, but in the second year it decreased in value by 10%. Complete the following to find the overall percentage change in the value of the investment over the 2 years.

Value after 1 year $= \frac{\square}{100} \times \$80\,000 = \$\underline{}$

Value after 2 years $= \frac{\square}{100} \times \$\underline{} = \$\underline{}$

Overall change in value over 2 years $= \$\underline{} - \$80\,000$

$$= \$\underline{}$$

Percentage change in value over 2 years $= \frac{\square}{80\,000} \times 100\%$

$$= \underline{}\%$$

10 A piece of antique jewellery was bought for \$2400. In the first year of ownership its value increased by 15%. In the second year it increased in value by a further 8%.
a Calculate its value at the end of the first year.
b Calculate its value at the end of the second year.
c What is the overall change in its value after 2 years?
d Find the percentage change in value over the 2 years.

11 An antique watch bought for \$15 000 increased in value by 5% in the first year of ownership and increased by 8% in the second year.
For the 2 years of this investment, calculate the:
a overall change in value
b percentage change in value.

WORKED EXAMPLE 4

a John's weight increased from 64 kg to 68 kg. Find the percentage increase in his weight.

b The value of a car decreased from $18 500 to $14 900 in 1 year. Calculate the percentage decrease in value.

Solve	Think	Apply
a *Method 1* Percentage increase $= \dfrac{68-64}{64} \times 100\%$ $= 6.25\%$	*Method 1* Increase $= 68 - 64$ kg $= 4$ kg Percentage increase $= \dfrac{4}{64} \times 100\% = 6.25\%$	*Method 1* Find the increase in weight and express this as a percentage of the original weight. Percentage increase $= \dfrac{\text{increase in quantity}}{\text{original quantity}} \times 100\%$
Method 2 $\dfrac{68}{64} = \dfrac{68}{64} \times 100\%$ $\quad = 106.25\%$ Percentage increase $= 106.25\% - 100\%$ $= 6.25\%$	*Method 2* $\dfrac{68}{64} \times 100\% = 106.25\%$ The increased weight is 106.25% of the original weight. Percentage increase $= 106.25\% - 100\% = 6.25\%$	*Method 2* Express the second weight as a percentage of the first and then subtract 100%. Percentage increase $= \dfrac{\text{second quantity}}{\text{first quantity}} \times 100\% - 100\%$
b *Method 1* Percentage decrease $= \dfrac{18\,500 - 14\,900}{18\,500} \times 100\%$ $= 19.5\%$ (to 1 decimal place)	*Method 1* Decrease $= \$18\,500 - \$14\,900$ $= \$3600$ Percentage decrease $= \dfrac{3600}{18\,500} \times 100\% \approx 19.5\%$	*Method 1* Find the decrease in value and express this as a percentage of the original value. Percentage decrease $= \dfrac{\text{decrease in value}}{\text{original value}} \times 100\%$
Method 2 $\dfrac{14\,900}{18\,500} = \dfrac{14\,900}{18\,500} \times 100\%$ $= 80.5\%$ (to 1 decimal place) Percentage decrease $= 100\% - 80.5\%$ $= 19.5\%$ (to 1 decimal place)	*Method 2* $\dfrac{14\,900}{18\,500} \times 100\% \approx 80.5\%$ The decreased value is 80.5% of the original value. Percentage decrease $= 100\% - 80.5\% = 19.5\%$	*Method 2* Express the second value as a percentage of the first and then subtract this result from 100%. Percentage decrease $= 100\% - \dfrac{\text{second quantity}}{\text{first quantity}} \times 100\%$

12 a Complete the following to find the percentage increase in weight from 52 kg to 55 kg.

Increase in weight $= 55 -$ _____ kg $=$ _____ kg

Percentage increase $= \dfrac{\square}{\square} \times 100\% =$ _____% $=$ _____% (to 1 decimal place)

b Complete the following to find the percentage decrease in cost from $186 to $154.

Decrease in cost $= \$$_____ $- \$$_____ $= \$$_____

Percentage decrease $=$ _____ $\times 100\% =$ _____% $=$ _____% (to 1 decimal place)

13 Find the percentage increase (to 1 decimal place) from:

  a $350 to $425   **b** 7.2 m to 7.8 m   **c** 63 kg to 68 kg

  d $80 to $215   **e** 4.2 kg to 8.4 kg   **f** 480 mL to 530 mL

14 Find the percentage decrease from:

  a $256 to $190   **b** 55 kg to 51 kg   **c** 15.8 s to 15.5 s

  d 430 ha to 385 ha   **e** $5400 to $1800   **f** 13 s to 12.2 s

INVESTIGATION 2.2

2J The unitary method

The unitary method is used to find the value of a number of items (or variables) by first finding the value of one item.

WORKED EXAMPLE 1

The cost of 3 kg of fish is $27.87. What is the cost of 5 kg of this fish?

Solve	Think	Apply
5 kg cost = ($27.87 ÷ 3) × 5 = $46.45	As the cost of 3 kg is given, find the cost of 1 kg by dividing by 3. Then find the cost of 5 kg by multiplying this result by 5.	First find the cost of 1 unit (in this case 1 kg) and then multiply by the required number of units 5 (kg).

EXERCISE 2J

1   The cost of 5 pairs of socks is $18.75. Find the cost of 6 pairs of these socks.

2   If 8 kg of potatoes cost $19.92, what is the cost of 5 kg of potatoes?

3   The cost of 50 m of rope is $63. Find the cost of 20 m of this rope.

WORKED EXAMPLE 2

A discount of 17.5% on a refrigerator results in the price decreasing by $152.25. Find the original price of the refrigerator.

Solve	Think	Apply
Original price = ($152.25 ÷ 17.5) × 100 = $870	17.5% of the original price = $152.25 1% of the original price = $152.25 ÷ 17.5 = $8.70 ∴ 100% of the original price = $8.70 × 100 = $870	Find 1% of the original price, then multiply by 100 to find (100% of) the original price.

4   A discount of 18% on a bicycle results in the price decreasing by $179.82. Complete the following to find the original price of the bicycle.

        18% of the original price = $____

         1% of the original price = $____ ÷ ____ = $____

    ∴ 100% of the original price = $____ × ____ = $____

5   At a certain school, 35% of the students travel by bus. If 259 students travel by bus, how many students are there at the school?

6   Linda and Matthew pay a 15% deposit of $84 000 on their new house. What is the total cost of the house?

7   It is calculated that 2% of the Australian fur seal population dies each year from entanglement in nets or ropes discarded by fishers. If approximately 600 seals die each year, estimate the size of the fur seal population.

WORKED EXAMPLE 3

A car dealer sells a car for $19 240. This represents the cost of the car to him plus a profit of 30%. For what price did the dealer buy the car?

Solve	Think	Apply
Cost price of car $= (\$19\ 240 \div 130) \times 100$ $= \$14\ 800$	130% of the cost price $= \$19\ 240$ 1% of the cost price $= \$19\ 240 \div 130$ $\qquad = \$148$ $\therefore$ 100% of the cost price $= \$14\ 800$	Find 1% of the cost price of the car and then multiply by 100 to find (100% of) the cost price.

8 A car dealer sells a car for $15 000. This represents a profit of 25% on his cost. Complete the following to find the cost of the car to the dealer.

  ____% of the cost price $= \$15\ 000$

   1% of the cost price $= \$15\ 000 \div$ ____ $= \$$____

 $\therefore$ 100% of the cost price $= \$$____ $\times$ ____ $= \$$____

9 Sarah's salary increase was 2.5%. If her weekly salary is now $697, what was her salary before the increase?

10 Ian has a 60% no-claim bonus on his car insurance.
 a If he receives a 60% discount on the full cost of the insurance, what percentage of the full cost does he pay?
 b If he pays $512, what is the full cost of the insurance?

11 This sector graph shows the results of an analysis of burglary claims made to an insurance company. In this particular year 1125 people were on holidays when their home was burgled.
 a How many were shopping?
 b How many were at work?
 c How many were on the premises?
 d How many claims were analysed by the insurance company?

Where were you when the burglar came?

Shopping 7%, At work 31%, On the premises 31%, Visiting 11%, Other 11%, On holidays 9%

12 The recipe shown for stir-fried pork serves 4 people.
 a How much of each ingredient would be needed to make this recipe for:
  i 6 people? **ii** 9 people?
 b Penny has 1.5 kg of pork. How many people can she feed using this recipe, assuming she has enough of all the other ingredients?

SPREADSHEET APPLICATION 2.1
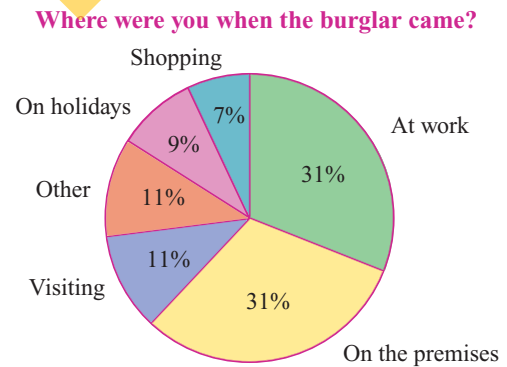
Stir-fried Pork

600 g pork
3 tbsp peanut oil
8 spring onions
$2\frac{1}{2}$ tsp grated ginger
2 tbsp lime juice
$\frac{1}{3}$ cup chicken stock
200 g sliced beans

MEASUREMENT
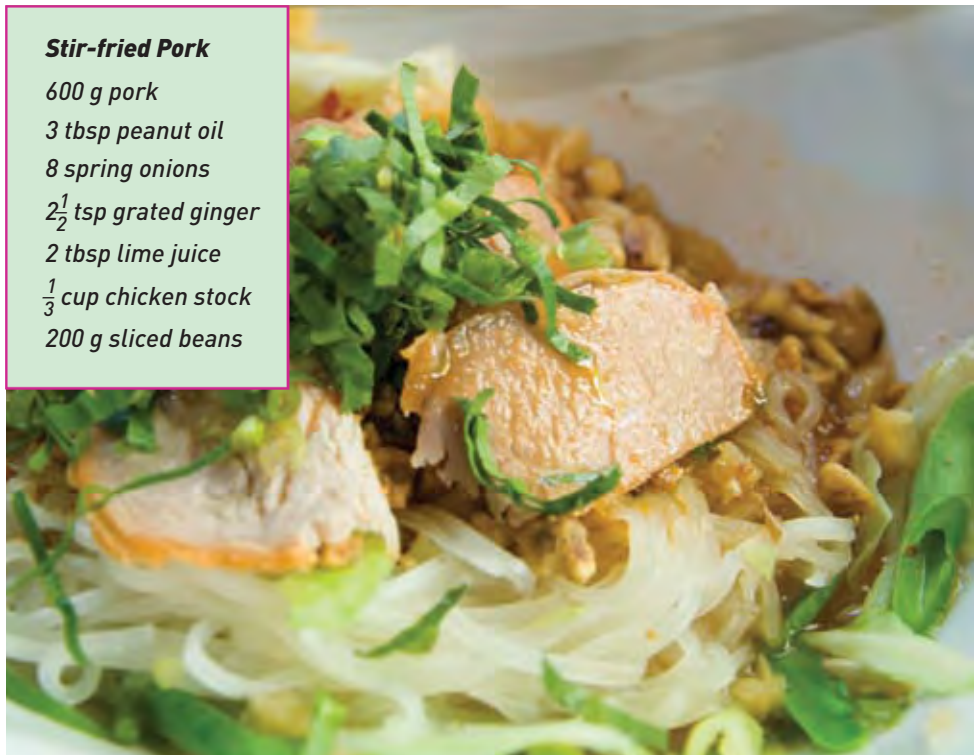
RESEARCH PROJECT **2.1**

On a map find the distance from Sydney to some other major cities of Australia. Calculate how long each journey would take by car, train and plane, by making reasonable assumptions about the (average) speed for each mode of transport. Write a report showing all your calculations and listing all assumptions made.

INVESTIGATION **2.1**

Modelling

1 A new car is bought for $30 000 and 8 years later its value is $5000. What would have been its value after 1, 2, 3, …, 7 years?

2 Can you predict its value after 9, 10, … years? You could consider the cases where the value:
- decreases by a fixed amount each year or
- decreases by a fixed percentage each year or
- you could make up your own model.

Check your models by researching the new and used prices of some cars. How do car dealers determine the values of used cars?

INVESTIGATION **2.2**

Calculate and compare freight costs for a variety of modes of transport. You could investigate the cost of sending a 10 kg parcel by truck, train or air transport from your town or city to another town or city. Vary the weight and size of the parcel. Vary the distance it is sent. Write a report on your findings.

SPREADSHEET APPLICATION **2.1**

Enter the quantities for four people for the recipe in question **12** Exercise **2J** into the cells of a spreadsheet and use it to calculate the quantities of each ingredient for different numbers of people. Print the resulting table with headings.

	A	B	D	E	F	G
1						
2			**Stir-fried Pork**			
3	**Ingredients**	**Unit of measurement**	**Serves 4**	**Serves 1**	**Serves 2**	**Serves 10**
4	Pork	grams	600	150	300	
5	Peanut oil	tablespoons	3	0.75	1.5	
6	Spring onions	units	8	2	4	
7	Grated ginger	teaspoon	2.5	0.625	1.25	
8	Lime juice	tablespoons	2	0.5	1	
9	Chicken stock	cups	0.33	0.0825	0.165	
10	Sliced beans	grams	200	50	100	

Language and terminology

1 Complete the statement: Scientific notation is also called _____ notation.

2 Scientific notation is useful for expressing what type of number?

3 List five possible sources of error in measuring.

4 What is the meaning of the term 'the limit of reading' of a measuring instrument?

5 Write down another term for 'the greatest possible error' in a measurement.

6 Explain how to determine the percentage error for a measurement.

7 What is the difference between a rate and a ratio?

8 What number is implied by the term 'unitary'?

Having completed this chapter

You should be able to:

- round numbers using significant figures
- express decimal numbers in scientific notation and vice versa
- perform calculations with numbers expressed in scientific notation
- convert between the commonly used metric units for length, mass, capacity and time
- understand the possible sources of error in measuring and how to reduce their effect
- determine the limit of reading, the greatest possible error, the upper and lower limits and the percentage error for a measurement
- find the maximum possible error when measurements are used in calculations
- make sensible approximations for the results of calculations using measurements
- calculate rates
- convert between units for rates
- solve problems involving rates
- simplify and find the ratio of two quantities
- divide a quantity in a given ratio
- increase and decrease a quantity by a given percentage
- determine the percentage increase or decrease in a quantity
- determine the overall change in a quantity after repeated percentage changes
- solve problems using the unitary method.

2 REVIEW TEST

1 When rounded to 2 significant figures, 3950.628 becomes:

    A 3900   **B** 4000   **C** 39   **D** 3950.63

2 Which of the following numbers is written in scientific notation?

    A $5 \times 10\,000$ **B** $50\,000$ **C** 5×10^4 **D** 50×10^3

MEASUREMENT

3 $7.06 \times 10^{-6} =$
   A 0.000 070 6   **B** 0.000 007 06   **C** 706 000   **D** 7 060 000

4 $(4 \times 10^5) \div (8 \times 10^{-3}) =$
   A 5×10^8 **B** 5×10^7 **C** 5000   **D** 5×10^2

5 5.06 kg =
   A 0.005 06 g   **B** 5060 g   **C** 0.0506 g   **D** 506 g

6 Which of the following is not equivalent to 5.3 m?
   A 530 cm   **B** 5300 mm   **C** 0.0053 km   **D** 0.053 km

7 The capacity of a drinking glass would be closest to:
   A 2 mL   **B** 20 mL   **C** 200 mL   **D** 2 L

8 3.3 h is equivalent to:
   A 3 h 30 min   **B** 3 h 3 min   **C** 3 h 20 min   **D** 3 h 18 min

9 6 ML is equivalent to:
   A 60 000 kL   **B** 6000 kL   **C** 600 kL   **D** 60 kL

10 The greatest possible error in the measurement 3.6 L is:
   A 0.1 L   **B** 0.05 L   **C** 0.5 L   **D** 3.55 L

11 The mass of a can of soup was 250 g to the nearest 10 g. The percentage error in this measurement is:
   A $\pm 4\%$ **B** $\pm 2\%$ **C** $\pm 0.4\%$ **D** $\pm 0.2\%$

12 The side length of a square was measured to be 8 cm. The maximum error in stating that the perimeter is 32 cm is:
   A 0.5 cm   **B** 2 cm   **C** 1 cm   **D** 4 cm

13 A garden hose can fill a 5 L bucket in 10 s. What is the rate of flow in litres per hour?
   A 180   **B** 1800   **C** 30   **D** 200

14 Water leaks from a tap at the rate of 3 drops per min. If 5 drops equal 1 mL, the amount of water wasted in a year is approximately (1 year = 365 days):
   A 105 L   **B** 7884 L   **C** 1577 L   **D** 315 L

15 If the current exchange rate is 101 US cents for each A$, how many Australian dollars would you need to exchange to get US$1000?
   A 900   **B** 1010   **C** 990.10   **D** 1000

16 The ratio of boys to girls in a school is 5 : 7. If there are 364 girls, then the number of boys is:
   A 260   **B** 55   **C** 275   **D** 77

17 When $56 000 is divided in the ratio 5 : 3 the larger part is:
   A $33 600   **B** $21 000   **C** $35 000   **D** $28 000

18 60 kg increased by 5% is:
   A 65 kg   **B** 5 kg   **C** 63 kg   **D** 3 kg

19 The percentage decrease from 80 kg to 74 kg is:
   A 92.5%   **B** 7.5%   **C** 94%   **D** 6%

20 What would be the value of a $30 000 painting after 2 years if it increases in value by 20% in the first year and then decreases in value by 20% in the second year?

    A $4800   **B** $24 000   **C** $28 800   **D** $30 000

21 If 6 loaves of bread cost $12.84, the cost of 10 loaves is:

    A $21.40   **B** $2.14   **C** $128.40   **D** $214

22 Melissa receives a 20% discount on a pair of jeans. If she pays $68 for the jeans, the amount she saved is:

    A $13.60   **B** $17   **C** $85   **D** $51

If you have any difficulty with these questions, refer to the examples and questions in the sections listed.

Question	1	2–4	5–8	9	10–12	13–15	16, 17	18–20	21, 22
Section	A	B	C	D	E	G	H	I	J

2A REVIEW SET

1 Round 3659.063 to:

    a the nearest 100   **b** the nearest whole number   **c** 2 decimal places

    d 3 significant figures   **e** 5 significant figures   **f** 1 significant figure.

2 State whether these numbers are written in scientific notation.

    a 6×1000 **b** 15×10^7 **c** 2.04×10^{-6}

3 Express these numbers in scientific notation.

    a 105 000 000   **b** 0.000 062

4 Calculate the following, writing the answers in scientific notation.

    a $(4.1 \times 10^8) \times (6 \times 10^5)$ **b** $(1.96 \times 10^{-3}) \div (1.4 \times 10^7)$

    c $(8 \times 10^5)^4$ **d** $\sqrt{8.41 \times 10^{-12}}$

5 Convert the following.

    a 5.6 cm² to mm²   **b** 43 000 m² to ha   **c** 2.9 m³ to cm³   **d** 5600 mm³ to cm³

6 Convert the following.

    a 2.3 Gm to m   **b** 52 ML to kL   **c** 3 ms to μs

7 Convert the following to minutes.

    a 3.5 h   **b** 3 h 50 min

8 Convert 258 s to:

    a minutes   **b** minutes and seconds.

9 For each of the following measurements, find:

    i the limit of reading

    ii the greatest possible error

    iii the lower and upper limits of the true measurement

    iv the percentage error (to 1 decimal place).

    a 7.5 m   **b** 280 g to the nearest 10 g

10 The length and breadth of a rectangle were measured to be 6 cm and 4 cm.

   a Calculate the perimeter using these measurements.

   b Write the lower and upper limits of the true length and breadth.

   c Find the lower and upper limits of its true perimeter.

   d Hence, what is the maximum error in the answer in part **a**?

   e Calculate the area using the measurements given.

   f Find the lower and upper limits of the true area.

   g Hence, find the maximum error in the answer to part **e**.

11 Write sensible approximations for the results of the following calculations.

   a 17.3 m + 15.89 m   **b** $17.3 \text{ m} \times 15.89 \text{ m}$

12 Convert the following.

   a 12 t/ha to kg/m²   **b** 9 m/s to km/h

13 The ratio of boys to girls in a school is 8 : 9. If there are 256 boys, how many girls are there?

14 Divide $48 000 into two parts in the ratio 3 : 5.

15 **a** Increase 80 kg by 3%.   **b** Decrease $290 by 15%.

16 **a** Find the percentage increase from $4.80 to $5.10.

   b Find the percentage decrease from 70 kg to 67.2 kg.

17 **a** What would be the value of a $25 000 vintage car after 2 years if it increases in value by 8% in the first year and then decreases in value by 5% in the second year?

   b Calculate the overall percentage change in value over the 2 years.

18 The cost of 8 pens is $6.32. What is the cost of 5 pens?

19 Penny received a discount of $25.70 on a mobile phone. This was 5% of the original price. Find the original price of the phone.

2B REVIEW SET

1 Round 1472.634 to:

   a the nearest 10   **b** 2 significant figures   **c** 2 decimal places.

2 Express the following in scientific notation.

   a 749 000   **b** 0.000 003

3 Calculate $(1.4 \times 10^7) \times (4.5 \times 10^8)$, expressing the answer in scientific notation.

4 Convert the following.

   a 2.1 ha to m²   **b** 780 mm² to cm²   **c** 9 500 000 cm³ to m³   **d** 72 cm³ to mm³

5 Convert the following.

   a 4.3 cm to µm   **b** 2 Tg to Mg   **c** 52 000 kL to ML

6 Write sensible approximations for the results of the following calculations.

   a 15.36 m + 9.7 m + 11.62 m   **b** $16.5 \text{ cm} \times 4.7 \text{ cm}$

MEASUREMENT

7 Sue buys 2.8 m of dress material for $99.68. What is the cost per metre of the material?

8 Water flows into a tank at the rate of 8 L/min.
  a How much water will flow into the tank in 3 h and 25 min?
  b If the tank has a capacity of 4.24 kL, how long will it take to fill the tank?

9 Simplify these ratios.
  a 36 : 45
  b $1\frac{1}{4} : 2$
  c 1.5 : 2.4

10 On one day at the school canteen, the ratio of bread rolls to sandwiches sold was 3 : 2 and the ratio of sandwiches to pies sold was 1 : 4. If the canteen sold 72 pies on this day, how many bread rolls were sold?

11 Kieren invests $8000 in a share fund. In the first year the fund increases in value by 12% and in the second year by 15%. Calculate the overall percentage change in the value of the fund.

12 The length of a table was measured to be 154 cm, to the nearest centimetre.
  a Write the limit of reading for this measurement.
  b What is the greatest possible error?
  c Determine the lower and upper limits of the true length.
  d Calculate the percentage error in this measurement.

13 If 6 dinner plates cost $29.34, what would be the cost of 8 dinner plates?

14 Julie's weekly wage increased to $494.40. If she received a 3% wage rise, what was her wage before the increase?

2C REVIEW SET

1 Round 0.005 06 to:
  a 2 significant figures
  b 2 decimal places.

2 Jin's weekly salary increased from $380 to $391.40. Calculate the percentage increase.

3 Convert the following.
  a 11.2 cm² to mm²
  b 129 000 m² to ha
  c 3.4 m³ to cm³
  d 73 000 mm³ to cm³

4 Convert the following.
  a 4.5 Mm to m
  b 2 Mg to t
  c 7 μs to ns

5 The weights of 2 bags of potatoes were measured, to the nearest kg, to be 49 kg and 51 kg.
  a What is the total weight of the 2 bags using these measurements?
  b Write the lower and upper limits of the true weight of each bag.
  c Calculate the lower and upper limits of the total weight of the 2 bags.
  d Hence, determine the greatest possible error in the answer to part **a**.
  e Express the greatest possible error as a percentage of the weight.

MEASUREMENT

6 Convert the following, giving the answer to 2 decimal places where necessary.

    a 60 km/h to m/s   **b** 50 g/m^2 to kg/ha

7 Write sensible approximations for the results of the following calculations.

    a 43.2 kg − 8 kg   **b** 125 g ÷ 85 mL

8 An amount of $15 000 is divided between two people in the ratio 5 : 3. What is the value of the smaller share?

9 The weight of meat decreases when it is cooked. If the ratio of the weight of raw meat to cooked meat is 1.25 : 1, calculate the weight of a 400 g piece of meat after it has been cooked.

10 Sean went on a diet and in the first week he lost 1.5 kg. This represented a 2% decrease in his weight. Calculate his weight at the start of the diet.

2D REVIEW SET

1 The number of children vaccinated at a clinic doubled from one year to the next. Comment on the statement: There was a 200% increase in the number of children vaccinated.

2 Round 2.0695 to:

    a 1 significant figure   **b** 2 significant figures

    c 3 significant figures   **d** 4 significant figures.

3 Convert the following.

    a 13.65 m to cm   **b** 3460 kg to t   **c** 276 s to min and s

4 Convert the following.

    a 13.65 ha to m^2 **b** 1960 mm^2 to cm^2

    c 3 700 000 cm^3 to m^3 **d** 6.8 cm^3 to mm^3

5 Convert the following.

    a 6 mL to μL   **b** 4.2 Gg to kg   **c** 8.1 μs to ns

6 Calculate the following, expressing the answer in scientific notation.

    a $(1.08 \times 10^{-6}) \div (7.2 \times 10^{-5})$ **b** $\sqrt{1.96 \times 10^{20}}$

7 The base and perpendicular height of a triangle were measured to be 15.4 cm and 12.5 cm.

    a Find the area of the triangle using these measurements.

    b Calculate the range within which the true area lies.

    c What is the greatest possible error in using part **a** as the area?

    d Express the greatest possible error as a percentage of the area.

8 A car uses 32 L of petrol to travel 250 km.
   a Calculate the petrol consumption in L/100 km.
   b At this rate of consumption, how much petrol (to the nearest litre) would be used to travel 650 km?
   c How far could the car travel on 56 L?

9 A photocopier is bought for $15 000. If it depreciates by 28% of its value each year, how long will it take for its value to fall below its scrap value of $1200?

10 Bill the builder purchased $2800 worth of materials from the hardware store. He receives a trade discount of 16% and then a further 5% discount if he pays within 3 days of receiving the account. What is the overall percentage discount if he pays within 3 days?

11 After a 15% discount, an LCD television costs $578. What was the original price?

2 | EXAMINATION QUESTION (15 MARKS)

   a Write 0.001 306 in scientific notation, rounded to three significant figures.   (2 marks)

   b The profits of a company are divided between the three directors in the ratio 2 : 2 : 3.
     If the company makes a profit of $630 000, calculate the largest share.   (1 mark)

   c The scale on a thermometer measures temperature to the nearest 0.5°C.
     i What is the greatest possible error in stating that the temperature is 19.5°C?   (1 mark)
     ii Within what range does the actual temperature lie?   (1 mark)

   d A large park of area 1.2 ha is to be fertilised at the rate of 14 g/m^2.
     i Calculate, in kilograms, the amount of fertiliser needed.   (2 marks)
     ii If the fertiliser is only available in 25 kg bags, how many bags are needed?   (1 mark)

   e An investment of $40 000 increases in value by 12% in the first year and decreases in value by 12% in the second year of investment.
     i Calculate the value of the investment at the end of the first year.   (1 mark)
     i Calculate the value of the investment at the end of the second year.   (1 mark)
     ii Find the overall percentage change in the value of the investment over these 2 years.   (1 mark)

   f A swimming pool measures 8.5 m by 5 m with an average depth of 1.4 m.
     i Calculate the volume of the pool in cubic metres.   (1 mark)
     ii If 1 L of water occupies 0.001 m^3, calculate the capacity of the pool in litres.   (1 mark)
     iii If water is supplied at the rate of 8.6 L/min, how long will it take to fill the pool?
        (Answer to the nearest hour.)   (1 mark)
     iv The owner begins to fill the pool at 11 am on Tuesday 10 November.
        At what time will the pool be full?   (1 mark)